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# Interactions stratégiques sur le marché d'une ressource non-renouvelable: Le cas du phosphore

Bocar Samba Ba

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# THÈSE

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Appliquée (LAMETA)

Spécialité : **Sciences Economiques**

Présentée par **Bocar Samba BA**

**Strategic Interactions on the Market of a Non-  
renewable Resource : The Phosphorus Case**

**Interactions Stratégiques sur le Marché d'une  
Ressource Non-renouvelable : Le Cas du  
Phosphore**

Soutenue le 01 Juillet 2016 devant le jury composé de

Mme Katheline SCHUBERT, Professeur des Universités,  
Université Paris 1 Panthéon-Sorbonne

Rapporteur et  
Présidente du jury

M. Stéphane LEMARIE, Directeur de Recherches,  
Institut National de Recherches Agronomiques de  
Grenoble

Rapporteur

M. Philippe MAHENC, Professeur des Universités,  
Université de Montpellier

Directeur de Thèse

M. Raphaël SOUBEYRAN, Chargé de Recherches,  
Institut National de Recherches Agronomiques de  
Montpellier

Co-directeur de Thèse

« L'Université n'entend donner aucune approbation ni improbation aux opinions émises dans cette thèse. Ces opinions doivent être considérées comme propres à leur auteur ».

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## Abstract

The theoretical literature that deals with phosphorus considers the market of the resource as being perfectly competitive, whereas the reality of this market suggests otherwise. Indeed, several interactions occur in this market. The main aim of this thesis is to rethink this market in an imperfectly framework. More specifically, we analyze the effect of recycling on the extraction of an exhaustible resource, on the dynamic of the resource price, on its date of depletion and on the reduction of water pollution. This thesis consists in a general introduction and five theoretical chapters all dealing with the economics of phosphorus or of exhaustible resources. **Chapter 1** considers a two-period model where an extractor and a recycler compete with quantities. We assume that extracted and recycled phosphorus are strategic substitutes. We show that the effect of recycling on the extracted quantities strongly depends on the level of the stock of phosphorus. **Chapter 2** extends the previous chapter in a continuous time framework over an infinite horizon. It investigates the effect of phosphorus recycling on the monopolist's extraction and on the dynamic of its price. We postulate an optimal control model and show that the price of the resource does not necessarily increase through time. **Chapter 3** considers that extraction and recycling can be either strategic substitutes or strategic complements. In a two-period model, we show that the effect of recycling on the monopolist's second-period marginal revenue and on its extracted quantities depends on whether extracted and recycled products are strategic substitutes or strategic

complements. **Chapter 4** considers that the extracting sector chooses between accommodating or preventing the recycler's entry. The entry prevention can take two forms: either deterring or blockading. In a two-period model, we show that the strategy of the extractor depends on the level of the fixed costs incurred by the recycler and on whether the resource is scarce or not. **Chapter 5** addresses the problems of phosphorus exhaustion and water pollution. We consider one firm that extracts and recycles phosphorus. We investigate the influence of a tax-subsidy scheme. We show that a combination of these two instruments enables to reduce water pollution and to prolong the lifetime of phosphorus.

**Keywords:** Strategic Interactions, Recycling, Phosphorus.

## Résumé

La littérature théorique portant sur le phosphore considère que le marché de la ressource est parfaitement concurrentiel, alors que son fonctionnement montre, en réalité, qu'il en est autrement. En effet, plusieurs interactions stratégiques existent sur ce marché. L'objectif principal de cette thèse est de reconsidérer ce marché dans un cadre de concurrence imparfaite. Il s'agit, particulièrement, d'analyser l'effet du recyclage sur l'extraction d'une ressource épuisable, sur la dynamique du prix de la ressource, sur sa date d'épuisement et sur la réduction de la pollution aquatique. Cette thèse est organisée autour d'une introduction générale et de cinq chapitres théoriques qui s'intéressent tous à l'économie du phosphore ou des ressources épuisables. Le **premier** considère un modèle à deux périodes où un pays extracteur et un pays recycleur se concurrencent en quantités. Nous supposons que le phosphore extrait et le phosphore recyclé sont des substituts stratégiques. Nous montrons que l'effet du recyclage sur les quantités extraites par le monopole est très sensible au niveau des réserves qui sont détenues par ce dernier. Le **deuxième** chapitre est une extension en temps continu du premier à horizon infini. Il analyse l'effet du recyclage du phosphore sur l'extraction du monopole et sur la dynamique du prix de la ressource. Nous utilisons un modèle de contrôle optimal et montrons que le prix de la ressource n'augmente toujours pas au fil du temps. Le **troisième** chapitre considère que l'extraction et le recyclage peuvent être soit des substituts stratégiques, soit des compléments stratégiques. Il considère un modèle à deux périodes et montre que l'effet du recyclage sur la recette marginale de deuxième période du monopole et sur ses quantités extraites dépend de si les quantités extraites et recyclées sont des substituts ou des compléments stratégiques. Le **quatrième** chapitre montre que le détenteur de la ressource arbitre entre accepter l'entrée du secteur de recyclage et l'empêcher. La dernière stratégie prend deux formes: soit l'extracteur dissuade l'entrée,

soit il la bloque. Nous utilisons un modèle à deux périodes et montrons que la stratégie adoptée par le détenteur de la ressource dépend de la taille des coûts fixes du recycleur et du niveau de rareté de la ressource. Le **cinquième** chapitre s'intéresse aux problèmes d'épuisement du phosphore et de la pollution aquatique. Nous considérons une firme qui extrait et recycle le phosphore. Nous analysons le rôle de la combinaison d'une taxe et d'une subvention. Nous montrons que la combinaison de ces deux instruments permet de réduire la pollution et de prolonger la durée de vie du phosphore.

**Mots-clés:** Interactions Stratégiques, Recyclage, Phosphore.

## General introduction

«All living organisms need phosphorus<sup>1</sup>». Phosphorus, which comes mainly from phosphate rocks<sup>2</sup>, was discovered in 1669 by the German Henning Brandt and has been used as a fertilizer for the first time in 1840, after German Liebig<sup>3</sup> (Cordell and White, 2011). It has been, intensively<sup>4</sup>, used for the first time as a fertilizer after the end of the second world war. Due to the rapid growth of the world population estimated to 9.6 billions<sup>5</sup> of people by 2050 and food security concerns, phosphorus has become more than ever a priority. The research work produced in this dissertation thesis focuses on this resource and is motivated by several reasons. The first one relates to the fact that, in spite of its economic importance, phosphorus has attracted very little interest from policy makers (Cordell and White, 2011) and there is very little academic<sup>6</sup> research about this resource. The few papers which have investigated the economics of phosphorus have assumed that the market of phosphorus is perfectly competitive (Weikard and Seyhan, 2009; Seyhan and al., 2012), whereas the reality of its functioning suggests otherwise. The oligopolistic or the quasi-monopolistic<sup>7</sup> structure of this market induces strategic interactions that do not occur naturally in a competitive market. The supply is not atomistic on this market, due to the fact that there are very few suppliers<sup>8</sup>. Phosphate

<sup>1</sup><http://ar2011.phosagro.ru/eng/ingredients/phosphorus/>.

<sup>2</sup>Extracted in the form of rock, phosphate is rinsed, screened and enriched until a white sand is obtained. This white sand is essential to the production of phosphoric acid and phosphate fertilizers.

<sup>3</sup>That's Liebig who has discovered that phosphorus can be used as a fertilizer. Before him, fertilization was based on the application of manure, crop residus and human wastes (Cordell et White, 2011).

<sup>4</sup><https://www.youtube.com/watch?v=63UZ6ef8V7k>; (Cordell et White, 2011).

<sup>5</sup><http://www.un.org/apps/newsFr/storyF.asp?NewsID=30521#.VpPUe1JVbhA>.

<sup>6</sup>Empirically, several papers have dealt with recycling of phosphorus.

<sup>7</sup>In terms of reserves, one may think that Morocco is in a quasi-monopolistic situation since some studies state that it holds more than 85% of world phosphate reserves.

<sup>8</sup>These few suppliers face with several purchasers.

reserves are actually in the control of only a handful of countries which have the power to set prices. These countries include Morocco<sup>9</sup>, which holds<sup>10</sup> 50,000 millions tons of phosphate rocks, China 3,700 millions tons, Algeria 2,200 millions tons, Syria 1,800 millions tons, South of Africa and Jordan with each of them 1,500 millions tons, United States of America 1,400 millions tons, Russia 1,300 millions tons and Senegal 50 millions tons (Van Kauwenbergh, 2010). The European Union, except for Finland, does not hold any phosphate reserves and is dependent on imports, which come mainly from Morocco<sup>11</sup>, Tunisia, Jordan and Syria (Ridder and al., 2012). As regards actual extraction (or production), United States of America were, until recently on top of the list, despite the small size of their reserves. Since 2012, China has become the top producer and its production amounts to 43% of the world production, whereas that of the United States of America amounts to 14%, that of Morocco amounts to 13% and that of Russia<sup>12</sup> amounts to 5% (Jasinski, 2013).

The global market of phosphorus is thus heavily dominated by a small group of countries, which, with a small population with respect to the size of their phosphate

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<sup>9</sup>Note that 2% of Morocco's total reserves are located within Western Sahara, an independent territory occupied by Morocco since 1975 (Copper, 2014).

<sup>10</sup>Out of a total of 67,000 millions tons of phosphates, Morocco holds 74%, China 6%, Algeria and Syria 3%, each of them, Jordan 2% and the other countries share the other 12% (Jasinski, 2013). Note that there is no consensus about the size of the reserves held by the different countries. For instance, through chapter 1, IFDC (2010) states that Morocco and Western Sahara hold 85% of global phosphate reserves, China possesses 6% whereas United States of America have only 3% of world phosphate reserves.

<sup>11</sup>Rosemarin (2015) states that European Union imports in the following range: 33% come from Morocco, 13% from Algeria, 11% from Russia, 9% from Israel, 8% from Jordan, 7% from Syria and 2% from Senegal.

<sup>12</sup>The estimation of the production of Russia is given by Rosemarin et Jensen (2013).

reserves, are potential exporters. Top exporters include Morocco<sup>13</sup> for which, exports<sup>14</sup> are around 35 and 45% of world exports (Watson and al., 2014), Jordan and Syria (Rosemarin and Jensen, 2015). Due to a strong domestic demand, the United States of America no longer export phosphorus and import it from Morocco. China, is no longer a phosphorus exporter, and has applied in 2008 an external tariff of 135% on phosphorus exports in order to secure domestic demand. The implementation of this external tariff has induced a rise of the price of phosphorus up to a level never seen before, close to 450\$ per ton of phosphorus (Cordell and White, 2011). With a relatively low population, and then small needs in phosphorus<sup>15</sup>, Morocco exports almost all its phosphorus.

The second reason relates to the fact that phosphorus has a crucial role for humanity, particularly for agriculture. Indeed, 90% of global demand<sup>16</sup> for phosphorus is for food production (Cordell and al., 2009). Phosphorus is taken as the green gold<sup>17</sup> of modern agriculture, because combined with nitrogen and potassium, it enables to increase the arable soils fertility and, therefore, agricultural yields. It enables a rapid growth of the root system of plants<sup>18</sup>, a good rigidity of the plants and a precocity of the fruits

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<sup>13</sup>In this country which is the top holder of phosphate reserves, the phosphate industry is managed by the Cherifian Phosphates Office (CPO). Note that it is the bigger moroccan industry. Phosphates account for 19% of Moroccan exports ([http://www.lemonde.fr/afrique/video/2015/11/05/ocp-qui-se-cache-derriere-le-geant-du-phosphate-marocain\\_4803912\\_3212.html](http://www.lemonde.fr/afrique/video/2015/11/05/ocp-qui-se-cache-derriere-le-geant-du-phosphate-marocain_4803912_3212.html)). In this country, three mineral deposits are exploited: the deposit of Khourigba, that of Gantour and that of Boucraa which is within the Western Sahara. Three ports (those of Jorf Lasfar (close to Khourigba), Safi (close to Gantour) and El-Ayoum (close to Boucraa)) host the productions coming from these sites, by railroad way or by slurry pipeline which is 187 kilometres long and which connects the deposit of Khourigba and the port de Jorf Lasfar.

<sup>14</sup>To refine its strategy of conquering the world market of phosphorus, the Cherifian Phosphates Office (CPO), has opened offices in Brasilia, India and expects to build a factory in Gabon. The strategy underlying the implementation of a factory in Gabon consists of capitalizing on the gabonese gas which is crucial in the transformation of phosphate and of ensuring an african production of fertilizers ([http://www.lemond:/e.fr/afrique/video/2015/11/05/ocp-qui-se-cache-derriere-le-geant-du-phosphate-marocain\\_4803912\\_3212.html](http://www.lemond:/e.fr/afrique/video/2015/11/05/ocp-qui-se-cache-derriere-le-geant-du-phosphate-marocain_4803912_3212.html)).

<sup>15</sup>Morocco consumes only 0.5% of world consumption of phosphorus.

<sup>16</sup>The demand for phosphorus is predicted to increase by 50 – 100% by 2050 (Steen, 1998).

<sup>17</sup><https://www.youtube.com/watch?v=63UZ6ef8V7k>.

<sup>18</sup>The plants which are deficient in phosphorus have a dark foliage, red or have red stains, and a less abundant flowering.



(Bello, 2010). It fosters also resistance of the plants to winter destruction (Mullins and Hajek, 1997). Phosphorus deficiency in soils induces malformed fruits, unfulfilled seeds with a slow maturation which yields low agricultural crops (Bello, 2010). In spite of its importance, phosphorus has no substitute in agriculture (Cordell and White, 2011). In addition to being the driving force of the modern agriculture, phosphorus plays other roles in other domains<sup>19</sup>, because it is essential in every cell of living organisms (Mikelsen, 2014).

The third reason refers to the exhaustion of phosphorus. In fact, some projections suggest that phosphate reserves will be depleted in a near future. Cordell and al. (2009) argue that the world phosphate reserves will be exhausted in another 50 – 100 years. Vaccari (2009) estimates that phosphate reserves will run out in 90 years. Steen (1998) highlights that economically-exploitable reserves will be depleted in 60 – 130 years, whereas Van Kauwenbergh (2010), who is more optimistic, stresses that world phosphate reserves will be exhausted in 300 – 400 years. It is noteworthy to mention that, before the exhaustion of phosphorus, experts predict that a peak of phosphorus will occur in 2033 (Craswell and al., 2010). This peak corresponds to the point in time at which the global demand of phosphorus will exceed its global production. From that moment on, the quality of phosphorus will decline, its price will increase continuously, it will be very costly to extract phosphorus and the farmers will have difficult access to this resource. The prospect of phosphorus rarefaction brings up questions about the development of alternatives to its depletion (Pellerin and Nesme, 2012). Accordingly, many solutions have been identified. Rosemarin (2015) recommends improving the efficiency in the extraction process by improving the technology. Cordell and al. (2009) distinguish demand solutions and supply solutions. On the demand side, they recommend reducing

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<sup>19</sup>It is a key component of bones and teeth, can be found in the molecules of DNA (Deoxyribonucleic Acide) and RNA (Ribonucleic Acid) and is, accordingly, determinant in the growth of humans (Cordell and White, 2011).

the losses of phosphorus (see also Mikelsen, 2014) and increasing the price of phosphorus. The latter solution<sup>20</sup> has the merit to force farmers to use the resource efficiently by not applying more than enough phosphorus on arable soils. On the supply side, solutions such as research and development in order to discover new economically-exploitable reserves and recycling<sup>21</sup> of phosphorus (see also Rosemarin, 2015) have been proposed. In the present thesis, we focus on the latter solution, i.e. recycling<sup>22</sup> of phosphorus. It consists of converting phosphorus contained in wastewater, ashes of sewage sludge<sup>23</sup>, human and animal excreta<sup>24</sup>. In order to reduce the amount of waste sent into landfill, to satisfy the increasing demand of phosphate and in anticipation of strengthening the amount of phosphorus content in the effluents, several countries have implemented technologies<sup>25</sup> in

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<sup>20</sup>This solution enables not only to reduce wastage of phosphorus but also to avoid the plants being burned by excess of phosphorus.

<sup>21</sup>It is important to note that other resources like aluminum, copper, gold and zinc are recyclable.

<sup>22</sup>Note that recycling of phosphorus by chemical precipitation and by biological phosphorus removal started in 1950s, in response to the growing problem of eutrophication (Morse and al., 1998).

<sup>23</sup>Sewage sludge contains more than 95% of phosphorus which enters the wastewater treatment plant. For instance, the wastewater treatment plant in Stockholm (Sweden) enables to recover phosphorus up to 95%. The incineration of sewage sludge does not induce phosphorus losses by volatilization, and phosphorus remains in the ash. Thus, the potential of recovering phosphorus from the ash of sewage sludge is high (Cohen and al., 2011).

<sup>24</sup>Cordell (2005) states that human excreta (urine and faeces) are renewable and are sources of phosphorus which are available. She stresses also that, according to some studies made in Sweden and Zimbabwe, the nutrients contained in urine of one person suffices to reduce 50 to 100% of food needs. Combined with other organic sources such as manure and food wastes, the value of phosphorus contained in urine and faeces can replace the demand of phosphate rock.

<sup>25</sup>The known technologies are chemical precipitation of phosphorus which consists of an addition of iron, aluminum, magnesium and calcium ions in water to treat (See Morse, 1998), flush toilets which allow for recovering urine without faeces because both get separated. It consists of putting holes through the toilet bowl. The urine passes into the front part (the holes) and the faeces into rear part. A large tank is placed somewhere and permits to collect urine, via a pipe. The latter can be applied directly on the arable soils or can be treated by reactors and transformed in a struvite which can be used as a fertilizer.

order to recycle<sup>26</sup> phosphorus. These include European Union<sup>27</sup> countries such as Germany, France, Netherlands, Denmark, Sweden, Belgium, United Kingdom, Poland and Austria. Other countries like Canada, Japan, China, Australia and the United States of America also recycle phosphorus. It is noteworthy to mention that the potential of recycling depends on the source of phosphorus and varies from country to country. (Van Dijk, 2013) stresses that the phosphorus recycling potential in the European Union is 61, 13% if phosphorus is recycled from sewage sludge, and 95, 31% if phosphorus is recycled from bones. In Germany, the rate of phosphorus recycling from wastewater is 50%, and from sludge 90%, whereas it is 80% in Sweden and in France, 50% in Netherlands, 32, 5% in the United States, and 96, 78% in Australia (Van Enk and al., 2011; Cornel and Schum, 2009; Shu and al., 2005). In terms of prospects, it is expected that the phosphorus recycling rate in Netherlands will reach 90% (Schipper and al., 2001), and Germany will increase its rate of recycling (Cornel and Schum, 2009). Note also that many European countries want to make phosphorus recycling mandatory in a near future (Van Dijk, 2013). Such is the case of Switzerland, Sweden, Germany and Netherlands.

The fourth reason is that phosphorus pollutes. In fact, after ending up into water due to water run-off, soil erosion, agricultural and industrial effluents, phosphorus pollutes water and creates eutrophication. The latter can be defined as the growth of algae which depletes the oxygen level in water and can cause suffocation of aquatic animals<sup>28</sup>. The absence of oxygen also entails fish death, resulting in deterioration of water quality and

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<sup>26</sup>The different firms which recycle phosphorus are the following: phosnix (Japan), ASH DEC (Austria), DHV, Thermphos International BV and SNB (Netherlands), Ostara (Canada), Seaborn (Germany) and Ecophos (Dunkerque-France). For more details, see: [www.bafu.admin.ch/uw-0929-d](http://www.bafu.admin.ch/uw-0929-d) or <http://www.eco121.fr/ecophos-ancre-sa-production-de-phosphate-a%CC%80-dunkerque/>.

<sup>27</sup>Recycling of phosphorus contained in sewage sludge produced in Europe could replace 20 or 30% of European union imports ([http://www.recophos.org/c/mid,1371,The\\_Challenge/](http://www.recophos.org/c/mid,1371,The_Challenge/)).

<sup>28</sup><http://www.vedura.fr/environnement/pollution/eau-eutrophisation-ecosystemes-aquatiques>

a change of the color of water. Accordingly, the use of water becomes dangerous both for animals and for humans.

In connection with the motivations above, the objectives of this thesis are as follows. First, we aim at rethinking the global market of phosphorus in an imperfectly framework. Second, as stated above, we will investigate one or several solutions to the exhaustion of phosphorus. Third, we will explore solutions which help at reducing pollution caused by phosphorus. Fourth, we will explain why the price of exhaustible resources does not always follow an upward phase, as stated by Hotelling (1931), the precursor of the economics of exhaustible resources.

The present thesis is based on five theoretical chapters<sup>29</sup>. **Chapter 1** focuses on the role of the stock of phosphorus in the relationship between the monopolist's extraction and recycling. It shows that if the level of the stock is sufficiently small, the monopolist extracts the whole resource in the first-period and the extracted quantity does not depend on recycling. By contrast, if the level of the stock is intermediate, phosphorus is depleted over the two periods and the monopolist's optimal extracted quantities depend on the existence of recycling. In this situation, its second-period extraction decreases in recycling, whereas its first-period extraction increases in recycling. If the stock is sufficiently large, phosphorus is not exhausted over the two periods and the extracted quantities depend on the recycled quantity. Consequently, the monopoly's extracted quantities decrease with recycling. This chapter considers that both extracted and recycled phosphorus are strategic substitutes. **Chapter 2** extends the previous chapter in a continuous time framework over an infinite horizon. It investigates the effect of phosphorus recycling on the path of extraction of phosphate reserves of a monopoly, on the exhaustion date of phosphorus, on the dynamic of the price of the resource and

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<sup>29</sup>The first and the third chapters are written with Robert Lifran (INRA & LAMETA) and Raphaël Soubeyran (INRA & LAMETA), the second with Raphaël Soubeyran and the fourth with Philippe Mahenc (University of Montpellier & LAMETA).

on consumers'surplus. We postulate an optimal control model and show the following results. First, the price increases through time if the level of recyclability is low. Second, the price decreases then increases if the level of recyclability is high. Third, the higher the recyclability rate, the more extraction and the exhaustion date are delayed. Fourth, a higher recyclability rate leads to an increase in price in the short-run (a decrease of consumers'surplus in the short-run) while it decreases after. **Chapter 3** considers that extraction and recycling can be either strategic substitutes or strategic complements. It shows that the possibility of recycling has two effects on prior price of raw products: a "recycling capacity" effect and a strategic effect. The "recycling capacity" effect always increases prior price of raw material, i.e. the monopoly decreases its first-period production in order to limit recycling quantities. The strategic effect increases the prior price of raw materials (decreases their prior production) only if raw and recycled products are strategic complements, whereas it decreases prior price (increases the prior production) if they are strategic substitutes. We then use two illustrative examples to show that both effects may dominate and that the first-period production increases or decreases accordingly. Making a link with the case of Alcoa<sup>30</sup> and the analysis of the green paradox, our results show that earlier results from the literature may be reversed under some specified conditions. The previous chapters assume that the monopoly always accommodates recycling. **Chapter 4** considers that the owner of the resource may have an incentive to prevent entry of the recycler. It assumes that the extraction sector can be perfectly competitive or can be a monopoly. It shows that, when the sector of extraction is competitive, two cases arise. If the fixed costs incurred by the recycler are low, the sector of extraction accommodates recycling by increasing its first-period extraction. In contrast, if the fixed costs are high, the sector of extraction reduces its first-period extraction in order to encourage recycling. When the sector of extraction behaves as a monopoly, two

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<sup>30</sup>Aluminum company of America.

cases arise also. If the fixed costs beared by the recycler are low, the monopolist can either ignore recycling or deter it. Indeed, it ignores recycling when the decrease of the future price of the resource is sufficient to discourage recycling. To deter entry, the monopolist can increase its first-period extraction in order to reduce the future price of the resource. This discourages recycling. We show that entry deterrence is the best strategy for the sector of extraction. Conversely, if in addition of low fixed costs, the resource is too scarce that recycling cannot be avoided, the monopolist accommodates recycling and reduces its first-period extraction in order to soften the future competition by limiting recycling. We also show that the Hotelling's rule must be amended in the presence of recycling. **Chapter 5** analyzes the role of an environmental tax-subsidy scheme as an instrument for preserving phosphate reserves and for improving water quality by reducing eutrophication. We use a model where one firm (that can behave as a price-taker or as a price-maker) extracts and recycles phosphorus. We assume the presence of a benevolent government that regulates the market by taxing extracted phosphorus and subsidizing recycled phosphorus. First, we show that taxing extracted phosphorus and subsidizing recycled phosphorus postpones the depletion of the resource and reduces water pollution. Second, we show that, in the case where the firm behaves as a price taker, only a Pigovian tax is necessary and it enables to achieve the first-best. Conversely, if the firm is a price-maker, the combination of the two policies is needed. In this case, the tax is lower than the marginal social damage. Third, we show that the tax-subsidy scheme does not modify the overall production supplied by the producer. Fourth, we show that the structure of the market is determinant in the ways to set the rate of the subsidy.

The contributions of the present thesis can be summed up as follows. First, in contrast to the earlier literature, we show that the effect of recycling on the pace of extraction

of phosphorus is sensitive to the size of the reserves held by the monopoly. Second, this thesis calls into question the conclusion of Hotelling that the price of exhaustible resources increases over time. We show in this thesis that this conclusion does not hold when the recyclability of the resource is sufficiently high. Instead of following an upward phase, the price of the resources can decrease in the recycling rate. Third, to the best of our knowledge, this thesis is the only one which considers that extracted and recycled products can exhibit strategic complementarity. This consideration is important because it reverses earlier conclusions and enables to revisit the analysis of green paradox which has received special attention in the academic literature. Indeed, in this case, the second-period marginal revenue of the monopolist increases in recycling. Anticipating this, the monopoly increases its second-period production. This reduces, mechanically, its first-period production. Such a result is at odds with the result established within the context of the green paradox which states that the eventual presence of a future substitute tends to lead the monopolist or the incumbent firm to increase its current production. Fourth, this thesis is the first one, to the best of our knowledge, to consider that the extraction sector facing with a competitive fringe of recyclers can behave either as competitive sector or as a monopoly. In the relationship between extraction and recycling, it is the only one to have considered that the extractor does not always accommodate recycling. Indeed, one can easily imagine that the holders of the natural resource will use strategies which will enable them to exclude recyclers from the market and to receive then the whole rent deriving from the sale of the resource. Fifth, the consideration of the polluting nature of phosphorus enables, not only, policymakers that hold phosphate reserves to know how to reduce eutrophication, but also proposes alternative solutions to the announced depletion of the resource.

In the following lines, we will present, in details, these five chapters that make up the body of our thesis.



## Introduction générale

«Tout ce qui vit a besoin de phosphore<sup>31</sup>». Le phosphore, qui provient en grande partie des roches de phosphate<sup>32</sup>, fut découvert<sup>33</sup> en 1669 par l'allemand Henning Brandt et a commencé à être utilisé en tant qu'engrais en 1840, suite aux travaux de Liebig<sup>34</sup> (Cordell et White, 2011). Toutefois, ce n'est qu'à partir de la deuxième guerre mondiale que son utilisation intensive<sup>35</sup> comme engrais a débuté. En raison de l'explosion démographique estimée à 9,6 milliards<sup>36</sup> à l'horizon 2050 et des enjeux de sécurité alimentaire, le phosphore est devenu, plus que jamais, une priorité. Le travail de recherche mené dans cette thèse s'intéresse à cette ressource, et est motivé par plusieurs raisons. La première tient au fait que, nonobstant son importance économique, le phosphore a reçu très peu d'attention de la part des décideurs publics (Cordell et White, 2011) et a fait l'objet de très peu de publications théoriques<sup>37</sup>. Les quelques rares articles qui ont exploré l'économie du phosphore ont considéré le marché mondial de la ressource comme étant parfaitement concurrentiel (Weikard et Seyhan, 2009; Seyhan et al., 2012) alors que son fonctionnement montre, en réalité, qu'il en est autrement. La structure oligopolistique ou quasi-monopolistique<sup>38</sup> de ce marché fait émerger des interactions stratégiques que l'on ne retrouve pas naturellement sur un marché de concurrence parfaite. Aussi, faudrait-il mentionner le fait que l'offre n'est pas atomistique sur ce marché, compte

<sup>31</sup><http://ar2011.phosagro.ru/eng/ingredients/phosphorus/>.

<sup>32</sup>Extrait sous forme rocheuse, le phosphate est rincé, criblé, puis enrichi jusqu'à l'obtention d'un sable blanc indispensable à la fabrication de l'acide phosphorique et des engrais phosphatés.

<sup>33</sup>[https://fr.wikipedia.org/wiki/Hennig\\_Brandt](https://fr.wikipedia.org/wiki/Hennig_Brandt).

<sup>34</sup>C'est Liebig qui a découvert que le phosphore peut être utilisé comme engrais. Avant lui, la fertilisation des sols était basée sur l'épandage du fumier, des résidus des récoltes et des déchets humains (Cordell et White, 2011).

<sup>35</sup><https://www.youtube.com/watch?v=63UZ6ef8V7k>; (Cordell et White, 2011).

<sup>36</sup><http://www.un.org/apps/newsFr/storyF.asp?NewsID=30521#.VpPUe1JVbhA>.

<sup>37</sup>Empiriquement, beaucoup d'études ont porté sur le phosphore, et particulièrement sur son recyclage.

<sup>38</sup>En termes de détention de réserves de phosphate, l'on peut probablement penser que le Maroc est en situation de quasi monopole comme certaines études estiment qu'il détient plus de 85% des réserves mondiales de phosphate.

tenu du fait qu'il y a très peu d'offreurs<sup>39</sup>, car les réserves de phosphate ne sont détenues que par une poignée de pays qui fixent le prix qu'ils souhaitent. Parmi les détenteurs des roches de phosphate, figurent le Maroc<sup>40</sup> qui détient<sup>41</sup> 50 000 millions de tonnes de phosphate, la Chine 3 700 millions de tonnes, l'Algérie 2 200 millions de tonnes, la Syrie 1 800 millions de tonnes, l'Afrique du Sud et la Jordanie avec chacune 1 500 millions de tonnes, les Etats Unis d'Amérique 1 400 millions de tonnes, la Russie 1 300 millions de tonnes, et le Sénégal 50 millions de tonnes (Van Kauwenbergh, 2010). Les pays de l'Union européenne, à l'exception de la Finlande, ne détiennent pas de réserves de phosphate et dépendent des importations qui proviennent en grande partie du Maroc<sup>42</sup>, de la Tunisie, de la Jordanie et de la Syrie (Ridder et al., 2012). En termes d'extraction, les Etats Unis d'Amérique étaient, jusqu'à un passé récent, les plus grands extracteurs, malgré leurs faibles réserves. Depuis 2012, ils sont relégués au second plan par la Chine dont la production s'élève à 43% de la production mondiale alors que celle des Etats Unis s'élève à 14%, celle du Maroc à 13%, et celle de la Russie<sup>43</sup> à 5% (Jasinski, 2013).

Fort de ce constat, nous comprenons aisément que la structure du marché mondial est fortement dominée par un petit nombre de pays qui, ayant une population faible relativement à leurs réserves, sont potentiellement des exportateurs. Ainsi, on trouve un groupe de pays dominés statistiquement en termes d'exportations par le Maroc<sup>44</sup>

<sup>39</sup>Ces offreurs font face à une multitude de demandeurs.

<sup>40</sup>Notons que 2% des réserves de phosphate détenues par le Maroc se situent dans le Sahara, un territoire indépendant mais qui a été occupé par le Maroc depuis 1975 (Copper, 2014).

<sup>41</sup>Sur un total de 67 000 millions de tonnes de phosphore, le Maroc détient 74%, la Chine 6%, l'Algérie et la Syrie 3% chacune, la Jordanie 2% et le reste des pays se partage les 12% (Jasinski, 2013). Il faut noter qu'il n'y a pas de consensus sur la taille des réserves détenues par les différents pays. Par exemple, à travers le chapitre 1, l'IFDC (2010) stipule que le Maroc-Sahara Occidental détient 85% des réserves mondiales, la Chine possède 6% alors que les Etats-Unis en possède 3%.

<sup>42</sup>Rosemarin (2015) établit les importations de l'Union Européenne dans l'ordre suivant: 33% proviennent du Maroc, 13% de l'Algérie, 11% de la Russie, 9% de l'Israël, 8% de la Jordanie, 7% de la Syrie et 2% du Sénégal.

<sup>43</sup>L'estimation de la production de la Russie est donnée par Rosemarin et Jensen (2013).

<sup>44</sup>Dans ce pays qui constitue le plus grand détenteur de phosphate, l'industrie du phosphate est gérée par l'Office Chérifien des Phosphates (OCP). Notons qu'elle est la première industrie marocaine. Les phosphates constituent 19% des exportations marocaines.

qui a un taux d'exportations<sup>45</sup> qui tourne entre 35 et 45% des exportations mondiales (Watson et al., 2014), la Jordanie, et la Syrie (Rosemarin et Jensen, 2015). Du fait d'une demande intérieure très forte, les Etats-Unis n'exportent plus de phosphore et ont recours au phosphore marocain. La Chine n'en exporte plus non plus et a appliqué en 2008 un tarif extérieur de 135% sur les exportations du phosphore afin de satisfaire les besoins domestiques. L'application de ce tarif extérieur a provoqué une hausse du prix du phosphore jusqu'à un niveau jamais égalé, frôlant les 450\$ par tonne produite (Cordell et White, 2011). Avec une population relativement faible et donc des besoins en phosphore peu<sup>46</sup> élevés, le Maroc exporte une très grande part de son phosphore.

La deuxième raison tient au fait que le phosphore est très important pour l'humanité, en particulier pour l'agriculture. En effet, 90% du phosphore extrait est destiné à l'agriculture (Cordell et al., 2009). Le phosphore est considéré comme étant l'or<sup>47</sup> vert de l'agriculture moderne, car combiné<sup>48</sup> à l'azote et au potassium, il permet d'améliorer la fertilité des sols arables et d'augmenter, par voie de conséquence, les rendements agricoles. Il permet également aux plantes<sup>49</sup> un meilleur développement du système

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([http://www.lemonde.fr/afrique/video/2015/11/05/ocp-qui-se-cache-derriere-le-geant-du-phosphate-marocain\\_4803912\\_3212.html](http://www.lemonde.fr/afrique/video/2015/11/05/ocp-qui-se-cache-derriere-le-geant-du-phosphate-marocain_4803912_3212.html)). Dans ce pays, trois bassins miniers sont exploités: le site de Khourigba, celui de la région de Gantour et celui de Boucraa qui se trouve dans le Sahara Occidental. Trois ports (ceux de Jorf Lasfar (Près de Khourigba), de Safi (près de Gantour) et de El-Ayoum (près de Boucraa)) accueillent les productions provenant de ces sites, par voie ferroviaire ou par le pipeline à boues ("Slurry pipeline") qui est long de 187 km et qui relie le site de Khourigba au port de Jorf Lasfar.

<sup>45</sup>Pour affiner sa stratégie de conquête du marché mondial du phosphore, l'office chérifien des phosphates (OCP) a ouvert des bureaux au Brésil, en Inde, et l'installation d'une prochaine usine est prévue au Gabon. La stratégie qui sous-tend l'ouverture d'une usine au Gabon consiste à capitaliser sur le gaz gabonais qui entre dans la composition du produit et aussi pour s'assurer une production africaine d'engrais.

<sup>46</sup>La part de la consommation du Maroc en phosphore dans la consommation mondiale est de 0.5%.

<sup>47</sup><https://www.youtube.com/watch?v=63UZ6ef8V7k>.

<sup>48</sup>Chacun des engrais joue un rôle particulier dans la croissance des plantes. L'azote participe principalement au développement du feuillage et des parties aériennes des plantes. Les plantes qui en manquent se développent lentement et présentent un feuillage vert clair et jaunâtre. Tandis que le potassium sert à la circulation de la sève et à l'assimilation des éléments nutritifs par les plantes. Il permet d'améliorer leur résistance au gel, aux ravageurs et maladies. Il contribue également à l'amélioration de la qualité gustative des fruits.

<sup>49</sup>Les plantes qui manquent de phosphore présentent un feuillage foncé, rouge ou marqué de taches rouges, la floraison est peu abondante.

racinaire, une bonne rigidité de la plante et une précocité des fruits (Bello, 2010). Aussi, favorise-t-il la résistance à la destruction des plantes par l'hiver (Mullins et Hajek, 1997). La carence des sols en phosphore entraîne des fruits mal formés, des graines peu remplies avec une maturation lente qui conduit aux faibles rendements (Bello, 2010). En dépit de son importance, le phosphore n'a pas de substituts en agriculture (Cordell et White, 2011). En plus d'être le moteur de l'agriculture moderne, le rôle crucial du phosphore s'étend à tous les domaines<sup>50</sup> de la vie, car il est essentiel à toutes les cellules des organismes vivants (Mikelsen, 2014).

La troisième raison est liée à l'épuisement du phosphore. De fait, certaines projections ont prévu son extinction dans un futur très proche. Cordell et al. (2009) estiment que les réserves mondiales de phosphate pourraient être épuisées dans les 50 à 100 prochaines années. Un rapport de Vaccari de 2009 indique que les réserves de phosphate naturels seront épuisées dans 90 ans. Steen (1998) souligne que les réserves économiquement exploitables s'épuiseront d'ici à 60 – 130 ans alors que Van Kauwenbergh (2010), qui est plus optimiste, estime que les réserves mondiales seront épuisées dans 300 – 400 ans. Il faut noter qu'avant l'épuisement de cette ressource, les spécialistes prévoient qu'un pic du phosphore aura lieu en 2033 (Craswell et al., 2010). Ce pic correspond au moment où la demande mondiale du phosphore deviendra supérieure à sa production mondiale. À partir de ce moment, le phosphore va perdre en qualité, son prix va progressivement augmenter, il deviendra plus coûteux de l'extraire et les agriculteurs y accéderont difficilement. La perspective d'une raréfaction du phosphore amène aujourd'hui à s'interroger sur l'élaboration des alternatives à son épuisement (Pellerin et Nesme, 2012). C'est à cet effet que plusieurs solutions ont été proposées. Rosemarin (2015) préconise d'améliorer l'efficacité dans l'extraction et d'améliorer la technologie.

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<sup>50</sup>Il est une composante très importante des os et des dents, est présent dans les molécules d'ADN (Acide Désoxyribonucléique) et d'ARN (Acide Ribonucléique) et est, par conséquent, indispensable à la croissance des êtres humains (Cordell et White, 2011).

Cordell et al. (2009) distinguent des solutions relatives à la demande et celles liées à l'offre du phosphore. Du côté de la demande, ils préconisent de diminuer les pertes de phosphore (voir aussi Mikelsen, 2014), et d'augmenter les prix de la ressource. Cette dernière solution<sup>51</sup> a le mérite d'obliger les agriculteurs à gérer la ressource de façon plus efficace en n'épandant pas sur les sols arables plus d'engrais qu'il n'en faut. Du côté de l'offre, des solutions telles que la recherche et le développement en vue d'explorer de nouvelles réserves économiquement exploitables et le recyclage<sup>52</sup> du phosphore (voir également Rosemarin, 2015) ont été proposées. Dans cette thèse, nous retenons la solution ayant trait au recyclage<sup>53</sup>. Ce dernier consiste à tirer profit des quantités importantes de la ressource contenues dans les eaux usées, dans les cendres des boues d'épuration<sup>54</sup>, dans les excréments des animaux et des humains<sup>55</sup>. A la fois pour limiter le volume de déchets mis en décharge, pour répondre à la demande croissante en phosphate et en prévision d'un renforcement des normes sur la teneur en phosphore dans les effluents, beaucoup

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<sup>51</sup>Cette solution permet non seulement de diminuer le gaspillage de la ressource mais également d'éviter que les plantes soient brûlées par un excès de phosphore.

<sup>52</sup>Notons qu'il existe d'autres ressources comme l'aluminium, le cuivre, l'or et le zinc qui sont recyclables.

<sup>53</sup>Notons que le recyclage par traitement chimique et biologique a commencé en 1950 pour limiter le problème croissant de l'eutrophisation (Morse and al., 1998).

<sup>54</sup>Les boues d'épuration contiennent plus de 95% du phosphore entrant dans une station d'épuration. Par exemple, l'usine de traitement des eaux usées à Stockholme, en Suède, a une efficacité de récupération du phosphore de 95%. L'incinération des boues d'épuration ne provoque pas de grosses pertes de phosphore par volatilisation, et le phosphore reste dans la cendre. Ainsi, le potentiel de récupération du phosphore à partir de la cendre des boues est élevé (Cohen and al., 2011).

<sup>55</sup>Cordell (2005) signale que les excréments humains (urine et matières fécales) sont renouvelables et sont des sources de phosphore qui sont disponibles. Elle souligne également que, selon quelques études effectuées en Suède et au Zimbabwe, les nutriments contenus dans l'urine d'une personne sont suffisants pour produire 50 à 100% des besoins alimentaires. Combinée avec d'autres sources organiques comme le fumier et les déchets alimentaires, la valeur du phosphore dans l'urine et dans la matière fécale peut essentiellement remplacer la demande de roche phosphatée

de pays ont développé des technologies<sup>56</sup> pour recycler<sup>57</sup> le phosphore. C'est le cas de certains pays de l'Union européenne<sup>58</sup> comme l'Allemagne, la France, la Hollande, le Danemark, la Suède, la Belgique, le Royaume Uni, la Pologne, et l'Autriche. D'autres pays tels que le Canada, le Japon, la Chine, l'Australie et les Etats Unis recyclent également le phosphore. Il est important de noter que le potentiel de recyclage du phosphore dépend de la source à partir de laquelle le phosphore est recyclé et varie d'un pays à un autre. Van dijk (2013) souligne que le potentiel de recyclage du phosphore dans l'Union Européenne est de 61,13% si le phosphore est recyclé à partir des boues d'épuration et de 95,31% s'il est recyclé à partir des os. En Allemagne, le taux de recyclage à partir des eaux usées s'élève à 50%, et à 90% lorsque le recyclage provient des boues d'épuration, tandis qu'il s'élève à 80% en Suède et en France, à 32,5% aux Etats-Unis, et à 96.78% en Australie (Van Enk and al., 2011; Cornel and Schum, 2009, Shu and al., 2005). En terme de perspectives, il est prévu que le taux de recyclage de la Hollande atteindra 90% (Schipper and al., 2001) et que l'Allemagne augmentera son taux de recyclage (Cornel and Schum, 2009). Aussi, faudrait-il noter que beaucoup de pays européens veulent que le recyclage du phosphore soit obligatoire dans un futur proche (Van Dijk, 2013). C'est le cas de la Suisse, de la Suède, de l'Allemagne et de la Hollande.

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<sup>56</sup>Les technologies connues sont la précipitation chimique du phosphore qui consiste en une addition, dans l'eau à traiter, des ions de fer, d'aluminium, de magnésium et de calcium (voir Morse, 1998), et les toilettes de séparation qui permettent de récupérer l'urine sans les déchets secs. Il s'agit de trouser la cuvette et de faire passer l'urine à travers les trous. Un réservoir placé quelque part permet de récupérer l'urine seule. Cette dernière peut être appliquée directement sur les sols arables ou traitée par des réacteurs et se transforme en struvite qui peut être utilisée comme étant de l'engrais.

<sup>57</sup>Les différentes firmes qui recyclent le phosphore sont les suivantes: Phosnix (Japon), ASH DEC (Autriche), DHV, Thermphos International BV et SNB (Hollande), Ostara (Canada), Seaborne (Allemagne) et Ecophos (Dunkerque-France). Pour plus de détails, voir: [www.bafu.admin.ch/uw-0929-d](http://www.bafu.admin.ch/uw-0929-d) ou <http://www.eco121.fr/ecophos-ancre-sa-production-de-phosphate-a%CC%80-dunkerque/>.

<sup>58</sup>Le recyclage du phosphore contenu dans les boues d'épuration produites en Europe pourrait remplacer 20 ou 30% des importations de l'Union européenne ([http://www.recophos.org/c/mid,1371,The\\_Challenge/](http://www.recophos.org/c/mid,1371,The_Challenge/)).

La quatrième raison tient au caractère polluant du phosphore. En effet, lorsqu'il se retrouve dans les eaux via le ruissellement, l'érosion des sols, les rejets agricoles et les rejets industriels, le phosphore pollue le milieu aquatique et engendre son eutrophisation. Cette dernière se définit comme étant le phénomène d'asphyxie des écosystèmes aquatiques résultant de la prolifération des algues qui consomment tout l'oxygène nécessaire à la vie des écosystèmes<sup>59</sup>. L'absence de l'oxygène dans le milieu aquatique entraîne de fortes mortalités piscicoles. Il en résulte une détérioration de la qualité de l'eau et un changement de sa couleur. Son usage devient, par conséquent, risqué pour la faune et la flore.

En relation avec les motivations citées dans les lignes précédentes, les objectifs de cette thèse se déclinent selon les termes suivants. Il s'agit, en premier lieu, de reconsidérer le marché mondial du phosphore dans un cadre de concurrence imparfaite. En deuxième lieu, comme indiqué ci-dessus, nous chercherons à trouver une ou des solutions à l'épuisement du phosphore. Il s'agit, en troisième lieu, de trouver une solution à la pollution causée par le phosphore. En quatrième lieu, nous expliquerons pourquoi le prix des ressources naturelles épuisables ne suit toujours pas une phase croissante, comme l'a souligné Hotelling (1931) qui est considéré comme étant le précurseur de l'économie des ressources naturelles.

Cette thèse est construite autour de cinq chapitres théoriques<sup>60</sup>. Le **premier** met l'accent sur le rôle essentiel que joue le niveau du stock ou des réserves de phosphate détenu par le monopole dans la relation entre le recyclage et les quantités extraites par le détenteur de la ressource. Il montre que si le niveau du stock est très faible, le monopole extrait tout son phosphore à la première période et le recyclage n'a aucun impact

<sup>59</sup><http://www.vedura.fr/environnement/pollution/eau-eutrophisation-ecosystemes-aquatiques>

<sup>60</sup>Le premier et le troisième chapitres sont écrits avec Robert Lifran (INRA & LAMETA) et Raphaël Soubeyran (INRA & LAMETA), le deuxième avec Raphaël Soubeyran et le quatrième avec Philippe Mahenc (Université de Montpellier & LAMETA).

sur l'extraction. En revanche, si le niveau du stock est intermédiaire, le monopole extrait toutes ses réserves sur les deux périodes. Dans ce cas de figure, le recyclage a un effet négatif sur la quantité extraite à la deuxième période et un effet positif sur celle extraite à la première période. La baisse de la quantité de deuxième période est due à la substituabilité stratégique, entre le phosphore extrait et le phosphore recyclé, qui signifie que l'augmentation de l'un entraîne inéluctablement la diminution de l'autre. Etant donné que la ressource s'épuise sur les deux périodes, la baisse de la quantité de deuxième période conduit mécaniquement à l'augmentation de la quantité de première période. Lorsque le niveau des réserves est assez grand, le recyclage a toujours un effet négatif sur l'extraction de la deuxième période du fait de la substituabilité stratégique, mais a un effet négatif sur l'extraction de la première période. Le **deuxième** chapitre est une extension en temps continu à horizon infini du chapitre précédent. Il analyse l'effet du recyclage du phosphore sur le sentier d'extraction du monopole, sur la date d'épuisement du phosphore, sur la dynamique du prix de la ressource et sur le surplus des consommateurs. Nous utilisons un modèle de contrôle optimal et montrons les résultats suivants. Premièrement, si le taux de recyclage ou de recyclabilité du phosphore est bas, le prix de la ressource augmente au fil du temps. Deuxièmement, si le taux de recyclage est, en revanche, élevé, le prix de la ressource diminue puis augmente. Troisièmement, nous montrons dans ce chapitre, que plus le taux de recyclage est élevé, plus l'extraction du phosphore est différée dans le temps et plus la date d'épuisement de la ressource est prolongée. Quatrièmement, une augmentation du taux de recyclage conduit à une baisse du surplus des consommateurs dans le court terme, en raison de l'augmentation du prix de la ressource et à une augmentation de ce surplus dans le long terme, du fait d'une baisse du prix de la ressource. Le **troisième** chapitre considère que l'extraction et le recyclage peuvent être soit des substituts stratégiques, soit des



compléments stratégiques. Il montre que la possibilité de recycler entraîne deux effets sur le prix de première période: un effet de recyclage et un effet stratégique. L'effet de recyclage augmente toujours le prix de première période des produits extraits, c'est-à-dire que le monopoleur diminue son extraction de cette période en vue de limiter la possibilité de recyclage. L'effet stratégique augmente le prix de première période des produits extraits (diminue leur production de cette période) seulement si les produits extraits et les produits recyclés sont des compléments stratégiques, alors qu'il diminue le prix de première période de ces produits (augmente leur production de cette période) si les produits extraits et les produits recyclés sont des substituts stratégiques. Nous utilisons alors deux exemples illustratifs pour montrer que la production de première période augmente ou diminue selon que tel ou tel effet domine. En lien avec le cas d'Alcoa<sup>61</sup> et l'analyse du "green paradox"<sup>62</sup>, nous montrons que les résultats établis par la littérature antérieure peuvent être renversés sous certaines conditions. Les chapitres précédents considèrent que le monopoleur accepte toujours l'entrée du secteur de recyclage. Le **quatrième** chapitre suppose que le détenteur de la ressource peut avoir une incitation à empêcher l'entrée du recycleur. Il considère que le secteur d'extraction peut se comporter soit comme un secteur concurrentiel soit comme un secteur monopolistique. Il montre que, lorsque le secteur d'extraction se comporte comme une entreprise concurrentielle, deux scénarii se présentent. Si les coûts fixes supportés par le recycleur sont faibles, le secteur d'extraction s'adapte à l'entrée en augmentant la quantité qu'il extrait à la première période. En revanche, si les coûts fixes que le recycleur supporte sont élevés, le secteur d'extraction doit réduire son extraction de première période pour encourager l'entrée du recycleur. Dans le cas où le secteur d'extraction se comporte

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<sup>61</sup>C'est une société américaine de production d'aluminium.

<sup>62</sup>Le term "green paradox" peut être traduit comme étant le paradox vert.

comme un monopoleur, deux situations se présentent également. Si les coûts fixes supportés par le recycleur sont faibles, le monopoleur peut soit ignorer le recyclage en se comportant comme si ce dernier n'est pas rentable, soit le dissuader. En effet, il ignore le recyclage lorsque la baisse du prix futur de la ressource est suffisante pour décourager le recycleur à entrer sur le marché. Pour dissuader l'entrée, le monopoleur peut augmenter son extraction de première période en vue de faire baisser le prix futur de la ressource, ce qui n'incite pas le recycleur à entrer sur le marché. Nous montrons que la dissuasion est la meilleure stratégie pour le secteur d'extraction. En revanche, si en plus des coûts fixes faibles, la ressource est tellement rare que le recyclage ne peut pas être évité, le monopoleur s'adapte au recyclage et réduit son extraction de première période dans le but d'atténuer la concurrence future via la réduction du recyclage. Aussi, montrons-nous que la règle d'Hotelling doit toujours être amendée en présence du recyclage. Le **cinquième** chapitre analyse le rôle de la combinaison d'une taxe et d'une subvention dans la conservation des réserves de phosphate et dans l'amélioration de la qualité de l'eau, via la réduction de l'eutrophisation. Nous utilisons un modèle où une firme (qui peut se comporter soit comme une entreprise concurrentielle soit comme une entreprise monopolistique) extrait et recycle le phosphore. Nous supposons la présence d'un gouvernement bienveillant qui régule le marché en taxant le phosphore extrait et en subventionnant le phosphore recyclé. Premièrement, nous montrons que la combinaison de ces deux politiques contribue à prolonger la durée de vie du phosphore et à réduire la pollution aquatique. Deuxièmement, nous montrons que, si la firme se comporte comme une entreprise concurrentielle, seule une taxe pigouvienne est nécessaire et elle permet d'atteindre la solution de premier rang. En revanche, si la firme se comporte comme une entreprise monopolistique, il faut combiner les deux instruments, à savoir la taxe et la subvention. Dans ce cas, la taxe est moins élevée que le dommage marginal.

Troisièmement, nous prouvons que la combinaison des deux instruments ne modifie pas la production totale offerte par la firme. Quatrièmement, nous indiquons que la structure du marché est déterminante dans la manière de fixer le niveau de la subvention.

Les apports de cette thèse peuvent être résumés comme suit. Premièrement, contrairement à la littérature antérieure, nous montrons que l'effet du recyclage sur le sentier d'extraction du phosphore est fortement lié au niveau du stock détenu par le monopole. Deuxièmement, cette thèse remet en question la conclusion d'Hotelling selon laquelle le prix des ressources naturelles épuisables augmente au fil du temps. Nous montrons dans cette thèse que cette conclusion n'est pas vérifiée lorsque le taux de recyclage ou de recyclabilité de la ressource est élevée. Au lieu de suivre une phase ascendante, le prix des ressources peut diminuer avec l'augmentation du recyclage. Troisièmement, à notre connaissance, cette thèse est la seule à avoir tenu compte du fait que la ressource extraite et la ressource recyclée peuvent être des compléments stratégiques. Cette considération est importante dans la mesure où elle remet en cause les résultats antérieurement établis et permet de revisiter l'analyse de "green paradox" qui a reçu une attention particulière dans la littérature académique. En effet, dans ce cas de figure, la recette marginale de deuxième période du monopole augmente. Anticipant cette hausse, le monopole augmente sa production de deuxième période. Ce qui diminue, mécaniquement, sa production de première période. Un tel résultat est en porte-à-faux avec celui obtenu dans le cadre du green paradox qui stipule que l'éventuelle présence d'un substitut futur tend à pousser le monopole ou l'entreprise qui est déjà installée sur le marché à augmenter sa production présente. Quatrièmement, cette thèse est la première à considérer que le secteur d'extraction qui fait face à une frange concurrentielle de recycleurs peut se comporter soit comme une entreprise concurrentielle, soit comme un monopole. Aussi, dans la relation entre l'extraction et le recyclage, est-elle la seule à considérer que l'extracteur

ne s'adapte forcément pas toujours à l'entrée du recycleur. En effet, l'on peut aisément imaginer que les détenteurs de la ressource naturelle mettront toujours en place des stratégies qui leur permettraient de rester seuls sur le marché et de récupérer toute la rente liée à la vente de la ressource. Cinquièmement, la considération du caractère polluant du phosphore permet non seulement de donner aux décideurs, qui possèdent des réserves de phosphate, une idée de comment faire pour réduire l'eutrophisation mais également donne des solutions pour pallier l'épuisement annoncé de la ressource.

Dans les lignes qui suivent, nous exposerons, de façon détaillée, ces cinq chapitres qui constituent le corps de notre thèse.

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## CHAPTER 1

## The Effect of Recycling on the Pace of Phosphate Rock Extraction: The Crucial Role of the Level of the Stock

### Abstract

Phosphorus is essential for agricultural production and therefore plays a key role in the global production of food by ensuring soil fertility and increasing crop yields. The element phosphorus has no substitute in agriculture. It is extracted from limited reserves of rock phosphate that are expected to be depleted in a near future. One way to postpone their depletion is to recycle phosphorus contained in food waste, sewage. Using a simple two-period model, we focus on the effect of phosphorus recycling by potential competitors on the monopolist's extracted quantities. We show that the effect of recycling depends on the level of the stock of phosphorus. If the level of the stock is sufficiently small, the monopolist extracts the whole resource in the first-period and the extracted quantity does not depend on recycling. By contrast, if the level of the stock is intermediate, phosphorus is depleted over the two periods and the monopolist's optimal extracted quantities depend on recycling. In this situation, its second-period extraction decreases in recycling, whereas its first-period extraction increases in recycling. If the stock is sufficiently large, phosphorus is not exhausted over the two periods and the extracted quantities depend on the recycled quantity. Consequently, the monopolist's extracted quantities decrease with recycling.

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**Keywords:** Phosphorus, Recycling, Stackelberg competition, Strategic substitutability.

## 1.1. Introduction

The element phosphorus<sup>1</sup> underpins our ability to produce food. It ensures soil fertility and increases crop yields. Its deficiency in agricultural soils impairs agricultural productivity and jeopardises food security (Runge-Metzger, 1995). Phosphorus for fertiliser is extracted from limited reserves of rock phosphate and there is no alternative to depletion in the long-term (Weikard and Seyhan, 2009). Remaining global phosphate reserves are in the control of only a handful of countries, including Morocco, China, United States of America, Iraq, Algeria, Syria. Morocco and Western Sahara hold 85% of global phosphate reserves<sup>2</sup>, China possesses 6% whereas United States of America have only 3% of world phosphate reserves (IFDC, 2010). Several projections suggest that these reserves will be exhausted in a few years (Steen, 1998; Vaccari, 2009; IFDC, 2010, Seyhan and al., 2012) and the expected global peak<sup>3</sup> in phosphorus production is predicted to occur in 2033<sup>4</sup> (Cordell and al., 2009). In addition to this uneven distribution between countries that makes phosphorus a strategic resource, it has no substitute in crop growth and cannot be manufactured (Cordell and White, 2013). While the timeline of phosphorus scarcity is contested, there is consensus that recycling<sup>5</sup> of phosphorus is required (Weikard and Seyhan, 2009; Cordell and White, 2013). Phosphorus can be

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<sup>1</sup>Phosphorus is also used in detergents and other chemicals. According to van Enk and al., 2011, all living organisms depend on phosphorus.

<sup>2</sup>In absolute terms, these reserves are estimated in 2012 to 50 millions of tonnes for Morocco, 3.7 millions of tonnes for China and 1.4 millions of tonnes for United States of America (USGS, 2012). There is a debate between data published by USGS and IFDC. Those published by the latter are more optimistic (see van vuuren and al., 2012).

<sup>3</sup>A peak of phosphorus is a point after which demand will outstrip supply (see Smit and al., 2009). Note that the point in time it will occur is contested, because calculations are based on phosphate rock reserves only (not on resources): see Ridder and al., 2012.

<sup>4</sup>This evaluation is based on reserves which are estimated to 16 000 Millions of tonnes of phosphate. As IFDC has re-estimated world reserves to 60 000 Mt, the peak of phosphorus is expected to occur between 2051 and 2092 with a mean of 2070 (Cordell and al., 2011).

<sup>5</sup>Recycling is a process that returns some of the phosphorus which is contained in output goods back into productive use (seyhan and al., 2012).

recovered from sewage, ash, food waste<sup>6</sup>, sludge, human excreta<sup>7</sup>, garden waste, crop losses<sup>8</sup> (Van Vuuren and al., 2010) and manure (Cordell and White, 2013). Presently, most sewage treatment facilities in Europe and North America remove phosphorus. This has been typically done by precipitation with iron or aluminum<sup>9</sup>. Phosphorus is also being removed in Japan, in China, and in Australia<sup>10</sup>.

In connexion with the issue of the exhaustion of phosphorus, the purpose of this chapter is to explore the effect of recycling by a consumer country on the paths of extraction of the resource holder. In other words, it consists of investigating whether recycling increases phosphorus lifetime or not. To address this issue, we use a two-period model where two countries compete with the quantities<sup>11</sup> of phosphorus they sell. The producing country has the monopoly<sup>12</sup>: it extracts the resource and sells it over the two periods. In the second-period, the consumer country<sup>13</sup> recycles a part of phosphorus that it consumed in the first-period. Consequently, recycling introduces competition in the second-period. Moreover, we assume that extracted phosphorus and recycled phosphorus are strategic substitutes.

As a result, we show that the effect of recycling of phosphorus by the consumer country (the recycler) on the producing country's pace of extraction depends on the level of the stock of phosphorus (or, equivalently, on whether the resource is depleted over the

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<sup>6</sup>The percentage of food waste reused as fertilizer and soil conditioner is 15% (Cordell and al., 2009).

<sup>7</sup>Humanity produces around 3 million tonnes of phosphorus each year (Cordell and al., 2009). Urine from one person alone provides more than half the per capita phosphorus required to fertilize cereal crops (Drangert, 1998). It can be used directly as a fertilizer in a safe way if it is not mixed with faeces in toilets and by taking simple precautions (Cordell and al., 2009).

<sup>8</sup>Crop losses or crop residues such as straw, husks and stalks can be ploughed back into the soils after harvest, for their soil conditioning and fertilizer value. Around 40% of the 5 million tonnes of phosphorus in crop residues generated annually are currently reused as fertilizer (Smill, 2002).

<sup>9</sup>[www.ostara.com](http://www.ostara.com).

<sup>10</sup>[www.ostara.com](http://www.ostara.com).

<sup>11</sup>Countries compete "à la Stackelberg".

<sup>12</sup>Country 1 can be associated with Morocco as it has, almost, the monopoly of phosphorus.

<sup>13</sup>Country 2 can be associated to European Union as there are, almost, not phosphate reserves in its earths and it has implemented the technology of recycling .

two periods or not). It varies depending on whether the stock is low or high. If the level of the stock is sufficiently small, the producing country extracts the whole resource in period 1 and its first-period extracted quantity does not depend on recycling. By contrast, if the stock is intermediate, phosphorus is depleted over the two periods and the producing country's optimal extracted quantities are dependent on the recycled quantity. In this situation, its second-period extraction decreases in recycling, whereas its first-period extraction increases in recycling. If the stock is sufficiently large, phosphorus is not exhausted over the two periods and extracted quantities depend on recycled quantity. Consequently, the producing country's both extracted quantities decrease in recycling. The level of the stock or the resource's constraint plays, then, an important role. The intuitions underlying these results are widely explained through our three propositions.

It is noteworthy to mention that many studies have, empirically, highlighted the key role of recycling of phosphorus consisting of postponing its exhaustion. Dumas (2009) suggests that phosphorus can be recycled at a rate of 80% while Christian and al. (2012), more optimistic, state that it may be recovered at a rate of 90%. This recycling may help to avoid the depletion of the resource (Cordell, 2007). Besides, it can reduce phosphorus emissions into receiving waters (Neset and al., 2008).

By contrast, the issue of recycling of phosphorus is not widely explored through theoretical models. To the best of our knowledge, only some papers have, theoretically, dealt with recycling. Weikard and Seyhan (2009) investigate the impact of phosphorus recycling in developed countries on the distribution of the resource between developing and developed countries and on the depletion of global reserves. They focus their investigation on recycling in developed countries because the treatment of waste and wastewater generates phosphorus-rich wastes as by-products and consider, by contrast, in most developing countries, wastewater treatment is virtually absent which leaves little

scope for the implementation of phosphorus recovery from wastewater. In other words, they consider that rich countries have developed and implemented technologies to recycle phosphorus, whereas poor countries don't recourse to these technologies. They assume that all countries rely on the same stock of primary resource supplied by a mining competitive<sup>14</sup> industry located in very few exporting countries. It is also supposed that the demand for phosphorus differs between the two types of countries (rich countries and poor countries), due to the fact that rich countries have phosphorus-saturated soils while soils in poor countries are deficient in phosphorus. They find that poor countries benefit in the short and medium run from phosphorus recycling in developed countries by increasing their imports, because they benefit from a lower price path but face stronger competition for the resource in the long run. Also, future generations in both, rich and poor countries, benefit from recycling in the developed countries given the fact that more phosphorus resources are available for them. Seyhan and al. (2012)<sup>15</sup> have investigated whether recycling contributes to phosphorus lifetime or not. They have introduced recycling into a dynamic model as a resource augmenting technology that creates a secondary resource. They consider technological progress in extraction and a stock effect which renders extraction more expensive since the stock is depleted. They assume that marginal extraction costs change<sup>16</sup> over time, firstly due to technological progress and secondly due to a stock effect. Indeed, the stock effect reflects geological conditions, i.e. marginal extraction costs differ between sites and change with the degree of extraction. Also, they consider technological progress in the recycling industry so that the marginal recycling cost may fall over time. They also assume that extracted and recycled phosphorus are

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<sup>14</sup>In the equilibrium each mining firm is indifferent between conservation and extraction. Hence, the relative change in resource price ( $P_t$ ) equals the interest rate ( $r$ ). This is the standard Hotelling's rule in a competitive situation. Formally,  $\frac{\dot{P}_t}{P_t} = r$

<sup>15</sup>While our results depend on a general discounted factor  $\delta$ , they have given to  $\delta$  some values to determine the effect of recycling on extracted quantity ( $\delta = 0,02$  in a certain case and  $\delta = 0,03$  in another case).

<sup>16</sup>Marginal extraction costs are reduced with technological progress and with the stock.

perfect substitutes, like in Weikard and Seyhan (2012) and their is a minimum<sup>17</sup> consumption of phosphorus. Recycling is introduced at the initial time period (unlike our model where it is introduced after the initial period). They show that recycling delays the depletion of phosphorus in the sense that it reduces considerably extracted phosphorus at the beginning and at the end shifting, therefore, the phosphorus extraction peak more into future. The fact that they have introduced recycling at the initial time period may explain why it reduces the extraction at this period. Thus, at this period recycled phosphorus can replace extracted phosphorus in consumption as both resources are assumed to be perfect substitutes. Unlike these authors, we do not introduce recycling at the beginning of the extraction but at a subsequent period. This specification induces anticipation effects that lead the monopolist to increase its initial extraction in the situation where phosphorus is fully exhausted. Note that both papers assume that the market is competitive. Theoretical models incorporating recycling have much more been explored in the field of aluminum, through the case of Alcoa. Recall that Alcoa is an american company that had a monopoly on the production of aluminum. It possessed more than 90 percent of aluminum production capacities (Swan, 1980; Grant, 1999), thus exceeding the legal threshold of monopolisation (Beir et Girmens, 2009). The Supreme Court prohibited it to acquire aluminum plants, under antitrust law. The Court, in its evaluation of Alcoa market power, excluded the secondary aluminum which was recycled by a competitive sector and that competed with Alcoa (Gaskins, 1974). In its defense, Alcoa argued that if secondary aluminum is taken into account, its production represented only 64 percent of the market. But the Court considered the capacity of the firm to influence the supply of the secondary aluminum. This affair, called «one of the most celebrated judicial opinions at that time», has motivated many authors to check whether the Supreme Court is right or wrong. In other words, it consisted, for them,

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<sup>17</sup>This minimum consumption reflects the essentiality of phosphorus.

of investigating whether the existence of a competitive recycling sector affects Alcoa's market power or not. Through an optimal control model, Gaskins (1974) shows that the existence of a competitive recycling sector makes the things worse in the short run, in that the price set by Alcoa exceeds the pure monopoly<sup>18</sup> price. This means that Alcoa's production decreases in the short run. Gaskins' model had been called into question by Swan (1980) due to the sensitivity of its results to the demand growth rate. Swan shows that Alcoa's market power is not, substantially, affected by recycling activity. Martin (1982) focuses on several cases. He shows that Alcoa's price exceeds the marginal cost of aluminum. Therefore, it maintains the monopoly rents. Grant (1999) finds, through an optimal control model, that Alcoa's market power is not affected by recycling, because it was too costly to recycle many primary products. So, this will limit the secondary aluminum supply. The overall conclusion of this theoretical line of research is that, in spite of the existence of a competitive recycling sector, the monopolist (Alcoa) maintains its market power. In other words, long-run price of aluminum charged by Alcoa is not, substantially, affected by recycling while our model indicates that it increases it by reducing the monopolist's second-period production.

Alcoa's case has been empirically explored by Suslow (1986). She shows that Alcoa maintains its market power, in spite of the existence of the recycling sector, not due to the fact that it exerts an influence on the secondary production, as mentioned by the American Supreme Court or by some authors, but due to the residual demand elasticity. Other reasons are, first, secondary aluminum and virgin aluminum are not perfect substitutes; second the interval between first sales of the primary product and the actual recycling is very long.

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<sup>18</sup>Pure monopoly is characterized by a form of market where there is only one producer and there is no substitute to his product.



The remainder of the chapter is organized as follows. Section 1.2 will present the structure of the market of phosphorus. We postulate a two-period model in section 1.3. Section 1.4 provides the concluding remarks, all appendixes and proofs are relegated in section 1.5.

## 1.2. Market structure

The phosphorus world market could be characterized by a typology of countries according to the ratio of demand to reserves. We identify a first group of countries, with huge population and important phosphate reserves, like China and the United States of America. Then, at the opposite, we have countries without phosphate reserves but large population, like the European Union (except Finland) or Brazilia. Countries like Morocco or Syria have huge reserves, compared to their population, and in absolute terms. Considering strategic nature of phosphate reserves, countries of the first group usually did impose formal bans on phosphate exportation. As a consequence, international trade is made between countries of the third group, being suppliers, and countries of the second group, being importers. Note also the United States of America, members of the first group, import phosphorus from Morocco, which is a member of the third group.

Considering now the importance of reserves, one could remark that they are defined according to the criterium of economic exploitability. During the shoc of 2008 and 2009 on the market, driven by the boosting demand for biofuels, the choc on phosphorus prices revealed that the reserves of Morocco have been lifted at ten times the level before the crisis, placing it in a long term position of de facto monopolist. In the short term, Morocco is not in position to control the market, because countries with significant reserves are able to react to increased prices, and to afford quantities in order to satisfy

a growing demand. However, it is expected that Morocco will be the monopolist in the long term.

### **1.2.1. Current production, exports and imports**

Global phosphate rock production is estimated at 181 million metric of tonnes (mmt) in 2010 and at 191 mmt in 2011 (Jasinski, 1999). In 2012, they are estimated at 71 000 mmt (USGS, 2012). The major producers are China, the United States of America (USA) and Morocco, in 2011 (see Figure 1.1). Their production accounts for over two-thirds of global production. These three biggest producers are followed by Russia, Jordan, and Tunisia. Major firms are comprised of Morocco's-owned PCO (Phosphorus Cherifian Organism), USA company Mosaic, Russian-owned PhosAgro and the Chinese Yuntianhua Group (Ridder and al., 2012).

In terms of export, PCO is the biggest world's exporter. To maintain this position it announced in 2010 that it expects to double its production capacities from 28 to 55 mmt and triple its production of fertilizers from 36 to 100 mmt. The other major exporter countries are Jordan and Syria. PCO currently makes between 35 and 40% of global exports. Although United States of America and China are among the top producers, their domestic consumption eclipses their exports (see figure 1.4).

Considering the exports by region, Africa (including Morocco) dominates the world market of exports by exporting 53.65% of global exports. It is followed by West Asia which exports 27.7% (see figure 1.3).

Data for individual countries' imports are not available (Ridder and al., 2012 ). Figure 1.5 shows that South Asia, West Europe, East Asia, Latin America and North America have the largest share in global imports, with 22.7%, 16.2%, 14.5% , 10.2%, and 9.6%, respectively.

### 1.2.2. Future dynamic of the market of phosphorus

Van Kauwenberg (2010) establishes that the world production of phosphorus will decline over time. It will meet the demand around 2050. It is expected that Morocco's production will increase over time from 15% by 2010 to 41% by 2050, while the production of China and that of United States of America will fall respectively from 37% to 26% between 2010 and 2050 and from 15% to 10% in the same period. By 2100, Morocco will produce 80% of the global production. At this point in time, Morocco will strengthen its monopoly position by holding 88.6 % of global reserves (Cooper and al., 2011). The authors point out that USA and Chinese reserves will be mostly or completely depleted this century, and with them around half of world's current phosphate rock production.

### 1.2.3. Dynamic of the prices of phosphorus

After a long period of relative stability, phosphate prices have become unstable. Indeed, prices increased from 2007, reach their maximum in 2008 and decline until June 2010 before increasing again (for more details, see figure 1.6). The sharp price increase can be explained by growing demand for fertiliser for production of crop-and animal-derived food, biofuels as well as a sudden rise in oil prices in the summer of 2008 may have resulted in a panic-driven phosphate market (van Enk and al., 2011). The peak of prices observed in 2008 can be also explained by the fact that China had applied an export tariff of 135% to secure its domestic supply (Cordell and al., 2009). This behavior had drastically reduced its exports and had affected the dynamic of the prices of phosphorus on the world market by inducing a 800% price spike (Cordell and al., 2011).

### 1.3. The model

We consider a two-period model with two countries, a producing country and a consumer country. The producing country extracts and sells phosphorus to the consumer country. At each period  $t = 1, 2$ , the producing country extracts and sells a quantity  $q_t \geq 0$  of a stock of phosphorus,  $S \geq 0$ . At time  $t = 2$ , the consumer country recycles a share<sup>19</sup>  $\alpha \in [0, 1)$  of the quantity of phosphorus consumed in period 1,  $q_1$ . So,  $r = \alpha q_1$ , with  $r$  recycled phosphorus. The discount factor is  $\delta \in (0, 1)$ .

The inverse demand function is  $p(Q_t)$ , where  $Q_t$  is the total quantity of phosphorus sold at time  $t$ . For simplicity, we assume that the inverse demand function is linear, i.e.  $p(Q_t) = a - Q_t$ , where  $a$  can be interpreted as the size of the market, or the maximum price at which phosphorus can be sold, or also the choke price (Sweeney, 1992). Under the previous assumptions, the producing country's payoff from phosphorus extraction is

$$(1.1) \quad \max_{q_1, q_2} (a - q_1)q_1 + \delta(a - q_2 - \alpha q_1)q_2$$

$$(1.2) \quad \text{s.t. } q_1 + q_2 \leq S$$

We solve the problem using lagrangian method. The lagrangian is given by

$$(1.3) \quad L = (a - q_1)q_1 + \delta(a - q_2 - \alpha q_1)q_2 + \mu(S - q_1 - q_2)$$

Where  $\mu$  is the Lagrange multiplier associated with (1.2). The solution for the maximization problem stated in (1.3) yields the results summarized in the following propositions:

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<sup>19</sup>Only a proportion of the extracted quantity is recycled because it is technically impossible to recycle whole consumed phosphorus.

**Proposition 1.** *If the level of the stock of phosphorus is sufficiently small ( $0 \leq S \leq \frac{1-\delta}{2-\delta\alpha}a$ ), the producing country exhausts the resource in the first-period, meaning that  $q_1^* = S$ . Therefore, its second-period extraction is zero,  $q_2^* = 0$ . In this situation, the quantity that it extracts is independent on phosphorus recycled by the consumer country in the second-period.*

**Proof.** see Appendix C □

The upcoming propositions show that the resource is not depleted in the first period.

**Proposition 2.** *If the stock of phosphorus is intermediate ( $\frac{1-\delta}{2-\alpha\delta}a \leq S \leq \frac{4-\alpha(1+\delta)}{4-\alpha^2\delta}a$ ), the stock of phosphorus is exhausted over the two periods. The quantity extracted by the producing country in the second-period decreases in recycling, whereas its first-period extraction increases in recycling.*

**Proof.** see appendix A □

Proposition 2 can be explained as follows. The presence of recycling in the second-period reduces the producing country's second-period extraction, due to the strategic substitutability of extracted and recycled resources. Both competitors do not have the incentive to increase simultaneously their production to keep the price of the resource high. Anticipating the decrease of its second-period extraction, the producing country increases its first-period production, because that part of the resource which is not consumed in one period will be consumed in the other period, due to the fact that the resource is fully depleted over the two periods. This result may seem counterintuitive in the sense that the increase of the first-period quantity induces the decrease of the price of phosphorus in this period. Usually the monopolist reduces the quantity it puts in the market to keep higher price (this result does not hold in some cases: see Stiglitz, 1976).

In contrast to this proposition, the following shows that the resource is not constrained in the second period.

**Proposition 3.** *If the stock of phosphorus is sufficiently large ( $\frac{4-\alpha(1+\delta)}{4-\alpha^2\delta}a < S$ ), the resource is not depleted over the two periods and both quantities extracted by the producing country decrease in recycling.*

**Proof.** see appendix B □

Proposition 3 states that the producing country's second-period extraction decreases in recycling. This is due to the strategic substitutability of both resources. The decreasing effect of recycling on the producing country's first-period extraction can be explained by the fact that recycling activity of the consumer country is nurtured by phosphorus consumed previously and which is purchased from the producing country. To soften the future competition by reducing the possibility for the consumer country to recycle, the producing country decreases its first-period extraction.

#### 1.4. Concluding remarks

This chapter outlines how phosphorus is important for agricultural production, describes the dynamic of the global market of phosphorus and analyses the effect of recycling on monopoly's paths of extraction. First, it highlights that the global market is dominated by Morocco in terms of phosphate reserves and in terms of exports. Trends indicate that Morocco will strengthen its monopoly position in the future. We have postulated a two-period model where a producing country competes with a recycling country in the second-period. This specification has allowed us to explore the influence of recycling of phosphorus on both quantities extracted by the monopolist. We have shown that this effect depends on whether the stock of phosphorus is low or high. If

the level of the stock is sufficiently small, the monopolist extracts the whole resource in the first-period and its first-period extracted quantity does not depend on recycling. By contrast, if the stock is intermediate, phosphorus is depleted over the two periods and monopolist's optimal extracted quantities are dependent on the recycled quantity. In this situation, its second-period extraction decreases in recycling, whereas its first-period extraction increases in recycling. If the stock is sufficiently large, phosphorus is not exhausted over the two periods and the extracted quantities depend on the recycled quantity. Consequently, the monopoly's both extracted quantities decrease in recycling.

It would be interesting to extend this simple model. First, as our results rely on the hypothesis of strategic substitutability of extracted phosphorus and recycled phosphorus, it would be interesting to see if they will change if both resources are strategic complements. Second, our study is based on an assumption of a fixed phosphorus stock that seems unrealistic because some new estimates prove that the stock of Morocco increases from 5 700 (Smit and al., 2009) to 50 000 millions tonnes of phosphorus (USGS data, 2012). This can be explained by the discovery of new phosphate reserves due to new technologies. It would be useful to postulate a model, which considers that reserves may vary over time. Another challenge for the future consists of setting up an optimal control model which will enable us to observe the dynamic of the price of phosphorus, and to analyze the continuous effect of recycling on the date of depletion of the resource.

## Appendix

### Appendix A: Determining the optimal quantities (when the resource is exhausted over the two periods) and proof of proposition 2

The monopolist maximizes the following programme

$$(1.4) \quad \max_{q_1, q_2} (a - q_1)q_1 + \delta(a - q_2 - \alpha q_1)q_2$$

$$(1.5) \quad \text{s.t. } q_1 + q_2 \leq S$$

The langrangian of this programme is given by

$$(1.6) \quad L = (a - q_1)q_1 + \delta(a - q_2 - \alpha q_1)q_2 + \mu(S - q_1 - q_2)$$

Assume that  $q_2 > 0$  and  $q_1 > 0$ . Hence, the first-order conditions are given by

$$(1.7) \quad \frac{\partial L}{\partial q_1} = a - 2q_1 - \alpha\delta q_2 - \mu = 0$$

$$(1.8) \quad \frac{\partial L}{\partial q_2} = \delta(a - 2q_2 - \alpha q_1) - \mu = 0$$

$$(1.9) \quad \mu(S - q_1 - q_2) = 0 \text{ and } \mu \geq 0$$

► Assume that

$$(1.10) \quad \mu > 0$$

Then

$$(1.11) \quad q_2 = S - q_1$$



Replacing  $q_2 = S - q_1$  in equations (1.7) and (1.8) gives

$$(1.12) \quad (a - \alpha\delta S) + (\alpha\delta - 2)q_1 = \mu$$

$$(1.13) \quad \delta(a - 2S) + \delta(2 - \alpha)q_1 = \mu$$

Solving (1.12)=(1.13) in  $q_1$  yields:

$$(1.14) \quad q_1^* = \frac{a(1 - \delta) + \delta(2 - \alpha)S}{[2 - \alpha\delta + \delta(2 - \alpha)]}$$

Equation (1.13) shows

$$(1.15) \quad \mu = \delta(a - 2S) + \delta(2 - \alpha)q_1$$

As

$$(1.16) \quad \mu > 0$$

We have

$$(1.17) \quad \delta(a - 2S) + \delta(2 - \alpha) \frac{a(1 - \delta) + \delta(2 - \alpha)S}{[2 - \alpha\delta + \delta(2 - \alpha)]} > 0$$

Solving (1.17) in  $S$  yields:

$$(1.18) \quad S < \frac{4 - \alpha(1 + \delta)}{4 - \alpha^2\delta} a$$

We know that

$$(1.19) \quad q_2 = S - q_1$$

Replacing  $q_1$  by its value in the equation above yields the second-period quantity given by:

$$(1.20) \quad q_2^* = \frac{(2 - \alpha\delta)S - a(1 - \delta)}{2 - \alpha\delta + \delta(2 - \alpha)}$$

Since  $q_2 > 0$ , we have:

$$(1.21) \quad S > \frac{1 - \delta}{2 - \alpha\delta}a$$

The combination of (1.18) and (1.21) yields:

$$(1.22) \quad \frac{1 - \delta}{2 - \alpha\delta}a < S < \frac{4 - \alpha(1 + \delta)}{4 - \alpha^2\delta}a$$

**Appendix B: Determining the optimal quantities (when the resource is not depleted over the two periods) and proof of proposition 3**

Now assume that

$$(1.23) \quad S - q_1 - q_2 > 0$$

Then

$$(1.24) \quad \mu = 0$$

And we obtain the following system of equations

$$(1.25) \quad a - 2q_1 - \alpha\delta q_2 = 0$$

$$(1.26) \quad \delta(a - 2q_2 - \alpha q_1) = 0$$

By rewriting (1.25), we have

$$(1.27) \quad q_1 = \frac{1}{2}(a - \alpha\delta q_2)$$

Putting  $q_1$  in (1.26) yields the second-period optimal quantity given by:

$$(1.28) \quad q_2^* = \frac{2 - \alpha}{4 - \alpha^2\delta}a$$

(1.28) in (1.27) gives the first-period quantity:

$$(1.29) \quad q_1^* = \frac{2 - \alpha\delta}{4 - \alpha^2\delta}a$$

In this case,

$$(1.30) \quad q_1 + q_2 < S$$

Substituting  $q_1$  and  $q_2$  by their values in (1.30) and making some simplifications yield:

$$(1.31) \quad S > \left(\frac{4 - \alpha(1 + \delta)}{4 - \alpha^2\delta}\right)a$$

### **Appendix C: Proof of proposition 1**

Assume that  $q_2 = 0$  and  $q_1 > 0$ . Hence, the first-order conditions are as follows:

$$(1.32) \quad \frac{\partial L}{dq_1} = a - 2q_1 - \mu = 0$$

$$(1.33) \quad \frac{\partial L}{dq_2} = \delta(a - \alpha q_1) - \mu \leq 0,$$

$$(1.34) \quad \mu(S - q_1 - q_2) = 0 \text{ and } \mu \geq 0$$

Assume that  $\mu > 0$ , then  $S = q_1$  (as  $q_2 = 0$ , according to (1.34)), (1.32) and (1.33) can be rewritten as

$$(1.35) \quad \mu = a - 2S$$

And

$$(1.36) \quad -(1 - \delta)a + (2 - \delta\alpha)S \leq 0$$

As  $\mu > 0$ , (1.35) implies that

$$(1.37) \quad S \leq \frac{1}{2}a$$

(1.36) yields

$$(1.38) \quad S \leq \frac{1 - \delta}{2 - \delta\alpha}a$$

(1.37) and (1.38) give

$$(1.39) \quad S \leq \frac{1 - \delta}{2 - \delta\alpha}a$$

Because

$$(1.40) \quad \frac{1 - \delta}{2 - \delta\alpha}a < \frac{1}{2}a$$

#### Appendix D: proof of the signs of extracted quantities

(i) If the stock of phosphorus is intermediate, i.e.  $\frac{1 - \delta}{2 - \delta\alpha}a < S < \frac{\delta(4 - 3\alpha) + \alpha}{4 - \alpha^2\delta}a$ , then extraction accelerates:

$$(1.41) \quad \frac{\partial q_1^*}{\partial \alpha} = -\frac{\partial q_2^*}{\partial \alpha} = \frac{1}{2}\delta(1 - \delta)\frac{a - S}{(\delta - \alpha\delta + 1)^2} > 0$$

i.e., it increases in the first-period and decreases in the second-period.

(ii) If the stock of phosphorus is sufficiently large, i.e.  $S > \frac{4-\alpha(1+\delta)}{4-\alpha^2\delta}a$ , then extraction slows down:

$$(1.42) \quad \frac{\partial q_1^*}{\partial \alpha} = -a \frac{\delta}{(\alpha^2\delta - 4)^2} (\delta\alpha^2 - 4\alpha + 4) < 0$$

and,

$$(1.43) \quad \frac{\partial q_2^*}{\partial \alpha} = -\frac{a}{(\alpha^2\delta - 4)^2} (\delta\alpha^2 - 4\delta\alpha + 4) < 0$$

i.e., decreases in the first-period and increases in the second-period.

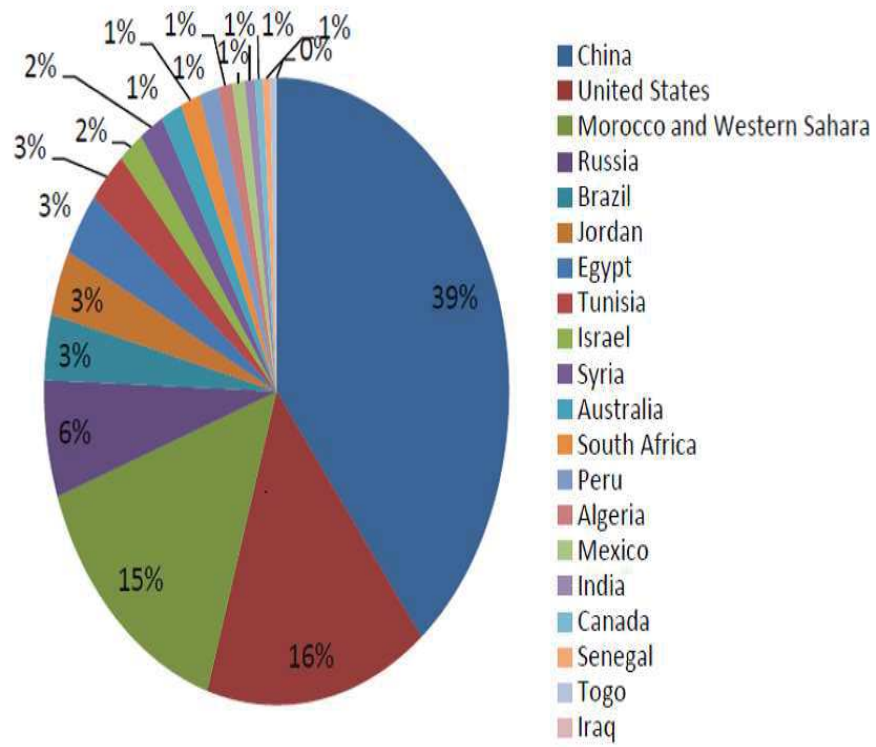


Figure 1.1. Phosphate production in 2011 (Jasinski, 2011)

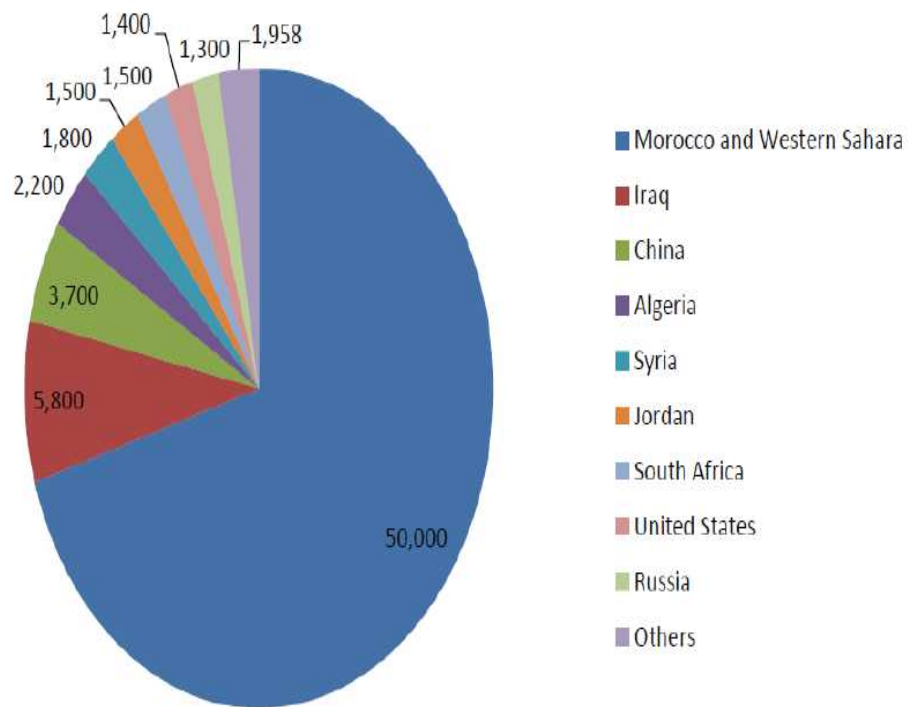


Figure 1.2. Phosphate reserves in absolute terms (Cooper and al., 2011)

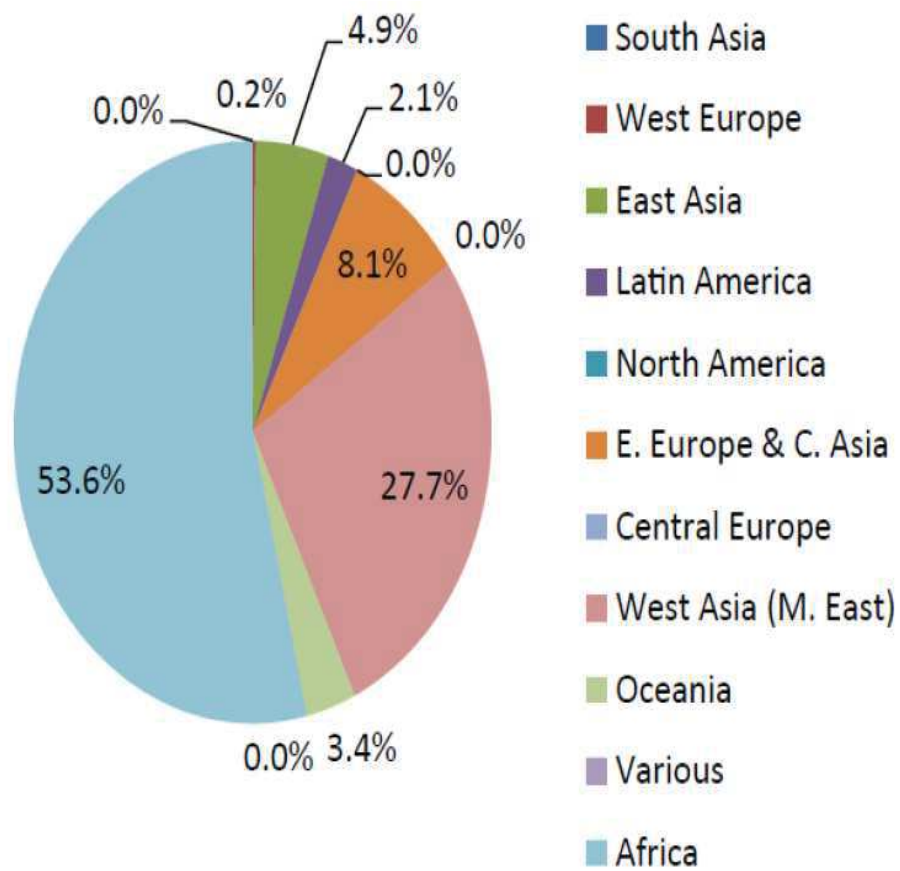


Figure 1.3. Phosphate rock exports by region (International Fertilizer Industry Association, "Statistics", 2011)



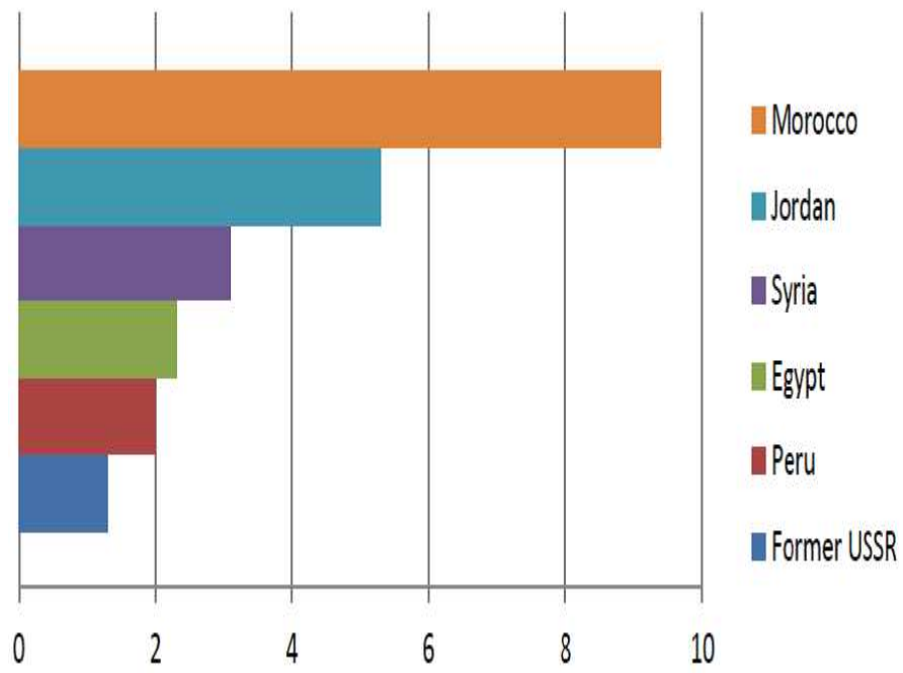


Figure 1.4. Phosphate rock exports by country (Ridder and al., 2012)

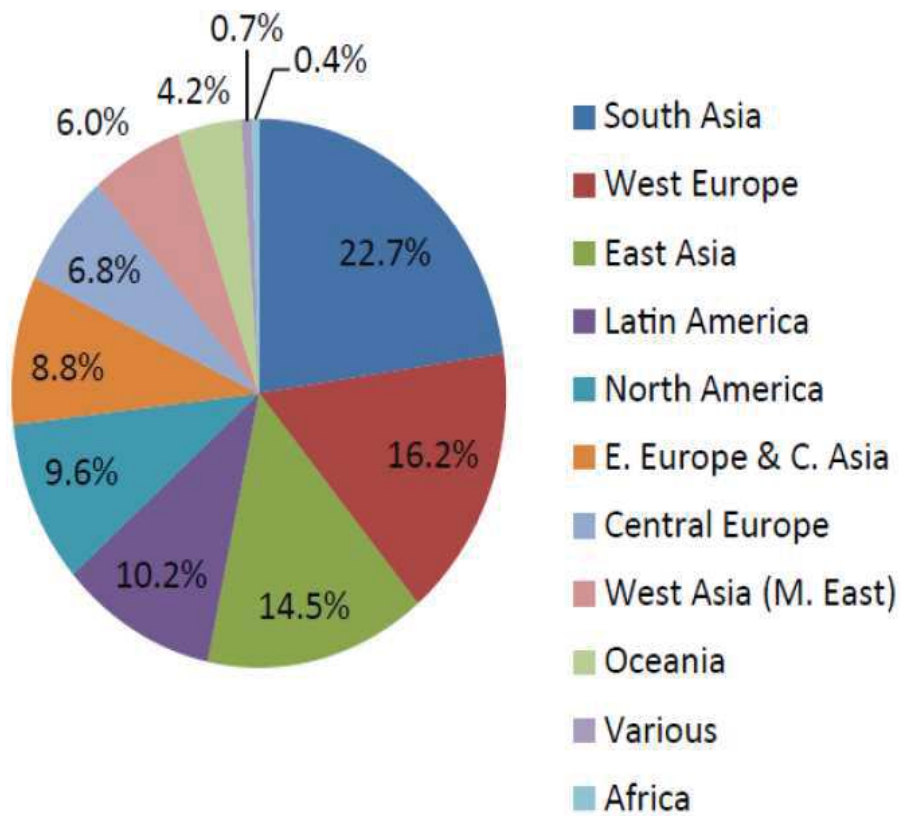


Figure 1.5. Phosphate rock imports by region (IFA, "Statistics")

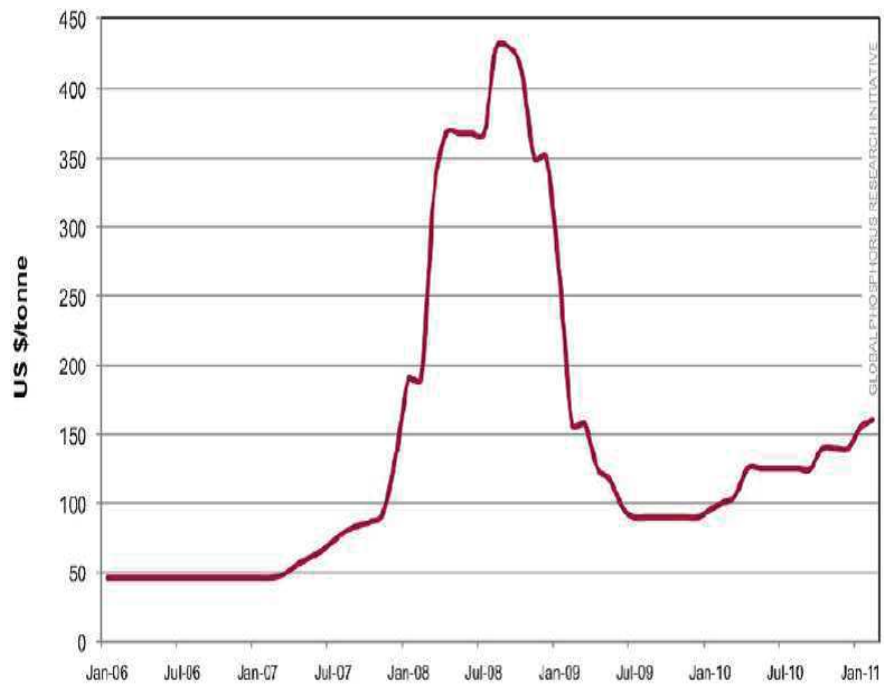


Figure 1.6. Phosphate rock commodity price (Cordell and White, 2011)

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## CHAPTER 2

# The Pricing of Recyclable and Exhaustible Resources: A Continuous Dynamic Model

## Abstract

We consider a dynamic model of extraction of an exhaustible resource by a monopolist, in which current demand increases future recyclable scrap. We analyze the effect of recycling on the rate of extraction of the monopoly, on the exhaustion date of the resource, on the dynamic of the price of the resource and on consumers' surplus. Using an optimal control model, we show four main results. First, the price increases through time if the level of recyclability is low. Second, the price decreases then increases if the level of recyclability is high. Third, the higher the recyclability rate, the more extraction and the exhaustion date are delayed. Fourth, a higher recyclability rate leads to an increase in price in the short-run (a decrease of consumers' surplus in the short-run) while it decreases after.

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**Keywords:** Pricing of Exhaustible Resources, Recycling, Optimal Control.

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## 2.1. Introduction

In the current context of the depletion<sup>1</sup> of phosphorus, recycling has attracted increasing attention. Indeed, many countries including Germany, Netherlands, Sweden, United States of America, Canada, China, Japan, etc. engage in recycling. In these countries, recycling has become a part of everyday life (Blomberg and Söderholm, 2009). Some studies argue that recycling will delay the exhaustion of this important resource (Cordell and al., 2009 and Cordell and al., 2011). To the best of our knowledge, only two papers have theoretically<sup>2</sup> dealt with the issue of recycling of phosphorus. Weikard and Seyhan (2009) and Seyhan and al. (2012) have investigated whether recycling contributes to the prolongation of the lifetime of phosphorus or not. Using optimal control models, both papers show that recycling delays the depletion of phosphorus. Also, Weikard and Seyhan (2009) show that recycling increases the short-run extraction, whereas it decreases the long-run extraction. Dealing with a continuous-time model, we show that the short-run extraction decreases in the recycling rate and the long-run extraction increases in the recyclability rate of the resource.

A number of interesting questions emerges in the present chapter. Does the price of phosphorus increase continuously over time, as suggested by Hotelling (1931) in the case of exhaustible resources ? Is recycling always beneficial to consumers ? Does recycling prolong the lifetime of phosphorus ? In the present chapter, we address these and related questions.

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<sup>1</sup>For instance, see Tweeten (1989): assuming that demand will increase at the rate of 3.6%, he stresses that phosphate reserves will be depleted in 61 years, i.e. in 2050; Smill (2000): he argues that phosphate reserves will be depleted in 80 years, i.e. in 2080; IDFC (2010): this report outlines that phosphate reserves will be exhausted in 300 – 400 years.

<sup>2</sup>Notice that there are many papers which have, empirically, explored the issue of recycling of phosphorus (see Cordell and al., ). In line with the theoretical papers, they have stressed that recycling delays the depletion of phosphorus.

In order to answer the questions above, we postulate an optimal control model where two firms compete. We consider a monopolist that holds phosphate reserves. Once it is consumed, phosphorus increases the stock of scrap. The monopolist is faced with a recycling sector that recycles one part of the stock of scrap. We assume that both firms do not bear costs.

We show that (i) the price of the resource increases through time if the level of recyclability is low, (ii) the price decreases then increases if the level of recyclability is high, (iii) the higher the recyclability rate, the more the extraction and the exhaustion date are delayed, (iv) a higher recyclability rate leads to an increase in price in the short-run (a decrease of consumers' surplus in the short-run) while it decreases after. It is worth pointing out that point (iv) results from the consideration of the continuous dynamic framework. Indeed, through a two-period model (Chapter 1), we have shown, in the case where phosphorus is exhausted over the two periods, that recycling increases the first-period production of the monopolist, meaning that it decreases the initial price of the monopoly. This contrasts sharply with point (iv).

This chapter is also related to another strand of the literature, which concerns the pricing of exhaustible or durable<sup>3</sup> products. Coase (1972) focused on the issue of the durable good monopolist. He argued that the monopolist of a durable good would be forced to provide the competitive equilibrium because the monopoly, having sold some output previously, would have the incentive to always maximize profits against the residual demand, and that this would entail producing more output so long as the total quantity was below the competitive quantity. Hence, the global stock would be close to the competitive stock, and the price would want to wait until the competitive

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<sup>3</sup>A durable good is a good that does not quickly wear out, or more specifically, one that yields utility over time rather than being completely consumed in one use. Phosphorus is taken as a durable good in the sense that it does not disappear completely, thank to the possibility of recycling. Other types of durable good include gold, diamonds, aluminum and silver and (Malueg and Solow, 1998; Levhari and Pindyck, 1981).

price was reached, forcing the monopolist to produce the competitive quantity right away. Knowing that the price of the durable good will decline, rational consumers prefer to postpone their purchase in the future. In order to incite consumers to buy in the present period<sup>4</sup>, some solutions have been identified. Indeed, the monopolist can rent the durable good instead of selling it (Bullock, 1982), or can find ways to commit not to increase the output in the subsequent periods. Focusing on a durable resource, Stewart (1980) stresses that the monopolist sets higher prices in the first-periods, whereas prices decline over time. Van den Berg and al. (2012) consider two asymmetric firms, which compete in quantity. They are asymmetric in the sense that one firm holds a large initial supply and the other firm has a medium-sized stock. They assume that the larger firm faces no capacity constraint, whereas the smaller firm is capacity-constrained in the second-period. They show that, in the absence of commitment, the equilibrium price strictly decreases over time. Gaskins (1974) finds, empirically, that the presence of a secondhand market can lead the monopolist to reduce its price. Indeed, he shows that the price charged by Alcoa is above the pure monopoly price for the first ten years of the planning period. In the tenth year, it equals the pure monopoly price and still decreases until reaching the equilibrium price. Istemi (2014) considers a resource duopoly model with two firms, competing in quantity for two consecutive periods. In the first-period, each firm is endowed with a fixed amount of exhaustible resource stock and is then allowed to invest in capacity in between the two periods of production in order to increase its resource stock. Thus, their second-period capacity constraints become endogenous. He finds that the equilibrium price weakly decreases over time with endogenous capacity constraints. In contrast to this earlier literature, we show that price decreases in short-run before rising. Contrary to the previously mentioned literature, some authors argue

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<sup>4</sup>Bond and Samuelson (1984) find that depreciation of the good and replacement sales reduce the monopolist's tendency to cut down price. Assuming constant marginal costs, Kahn (1986) echoes the conclusion of Bond and Samuelson (1984).

that the price of an exhaustible resource can follow an upward direction. Using a non-cooperative Cournot oligopoly model, Loury (1986) shows that aggregate production of oil decreases over time, meaning that its price increases over time. Van den Berg and al. (2012) find that, under commitment, the price weakly increases over time. Our result is consistent with theirs in the case where the recycling rate is low. We show that when the recyclability of the resource is high the price does not increase continuously, in the sense that the price curve is U-shaped.

The primary contribution of this chapter is that the price of an exhaustible resource can take a downward phase. This result is not consistent with Hotelling's conclusion, which states that the scarcity of rent of the exhaustible resources would cause the prices to increase over time (Hotelling, 1931). This result captures the stylized characteristics of exhaustible resource markets where price declines are, sometimes, observed.

To the best of our knowledge, there is not another paper which shows that the price of a recyclable resource can be U-shaped. Only Levhari and Pindyck (1981) show a similar result, but within the context of durable resources. There is a significant difference between durable and recyclable resources. In the case of a durable resource, the demand is a stock relationship, while it is a flow relationship in the case of a recyclable resource (Levhari and Pindyck, 1981, footnote 5). Their paper shows that the price always falls, and the price profile is U-shaped. The implication of the present chapter is that the price of an exhaustible resource can follow a downward phase, not due to the discovery of new reserves or the improvement of the technology of extraction, but thank to recycling.

Empirically, Martinez-de-Albéniz and Talluri (2011) investigate price competition for an oligopoly in a dynamic setting, where each of the sellers has a fixed number of units available for sale over a fixed number of periods. They assume that demand is stochastic and find that prices decrease in the first-periods and increase in the last

periods. Hnyilicza and Pindyck (1976) characterize the optimal price trajectories for a cartel (OPEC). For some discount rates<sup>5</sup>, they show that the price of the exhaustible resource decreases<sup>6</sup> (from 1975 to 1980) before rising slowly (from 1980 to 2010).

The remainder of the chapter is organized as follows. The next section introduces the model in which the monopolist of the exhaustible resource faces a recycling fringe. Section 2.3 studies the optimal recycled quantity. Section 2.4 describes the optimal extraction path and price dynamics. The role of recyclability is established in section 2.5. The main conclusions and some further research lines are given in section 2.6 and all proofs are relegated to the appendix.

## 2.2. The Hotelling Model with a Recycling Fringe

Consider an economy in which consumption is given by  $Q$ . The consumption good is a nonrenewable resource which can be recycled and the virgin and recycled materials are assumed to be perfect substitutes in demand. There is a unique resource owner and a competitive fringe of recycling firms.

**Nonrenewable resource and scrap dynamics:** The resource is characterized by its initial stock,  $X^0 \geq 0$ . Let  $X \geq 0$  be the residual stock at time  $t$  and  $x \geq 0$  the extraction rate. Then,

$$(2.1) \quad \dot{X} = -x.$$

Let the cost of extraction of the resource be zero. Let  $r \geq 0$  be the quantity of recycled materials marketed at time  $t$ . The total quantity consumed at time  $t$  is then  $Q = x + r$ . Let  $S(0) = S^0 \geq 0$  be the initial quantity of scrap. Let  $S \geq 0$  be the stock of scrap at time  $t$ . Let  $\alpha \in [0, 1]$  the proportion of resource that is not recycled that becomes

<sup>5</sup>For instance  $\delta = 0.02$ ,  $\delta = 0.05$ ,  $\delta = 0.10$ .

<sup>6</sup>Except for the initial date where the cartel charges higher price.

recyclable scrap. The dynamic of scrap writes  $\dot{S} = \alpha(Q - r)$  or,

$$(2.2) \quad \dot{S} = \alpha x,$$

where parameter  $\alpha$  represents the "recyclability" rate of the nonrenewable resource.

**The recycling sector:** The recycling sector is a competitive fringe. We assume that the marginal cost of recycling is decreasing in the stock of scrap net of the quantity of recycled materials (i.e. the remaining stock of scrap):

$$(2.3) \quad c(S, r) = 1 - b - \beta(S - r),$$

with  $\beta > 0$  and  $b \in (0, 1)$ .

We assume that the inverse demand is linear,  $p(Q) = 1 - Q$ . Thus,  $b$  is a measure of the added value of recycled material compared to scrap.

In equilibrium, the price must equal the marginal cost of recycling:

$$(2.4) \quad p(Q) = c(S, r).$$

**The extraction sector:** The owner of the resource chooses the optimal level of extraction that maximizes its discounted profits with discount rate  $\delta \geq 0$ ,

$$(2.5) \quad \underset{\{x\}}{Max} \int_0^{+\infty} e^{-\delta t} p(Q) x dt,$$

subject to (2.4), (2.1), (2.2),  $X, S, x \geq 0$  and  $X^0$  given and  $S(0) = 0$ .

### 2.3. Optimal Extraction of the Recyclable Resource

Solving the recycling sector equilibrium condition (2.4), we characterize the equilibrium quantity of recycled material at time  $t$  as follows:

$$(2.6) \quad r = \frac{b + \beta S - x}{1 + \beta}.$$

Thus, the quantity of recycled material at time  $t$  increases with the quantity of scrap and decreases with the quantity of extracted resource. We focus on the cases in which the right hand side is nonnegative.

The current value Hamiltonian  $H$  and Lagrangian  $\mathcal{L}$  are defined as follows:

$$(2.7) \quad H = p(Q)x + \lambda_X(-x) + \lambda_S(\alpha x),$$

and,

$$(2.8) \quad \mathcal{L} = H + \mu_X X + \mu_S S + \mu_x x,$$

where  $\lambda_X$  and  $\lambda_S$  are the co-state variables associated with the stocks  $X$  and  $S$ , and,  $\mu_X, \mu_S, \mu_x$  the multipliers associated with the nonnegativity constraints  $X \geq 0$ ,  $S \geq 0$  and  $x \geq 0$ . The total quantity writes  $Q = x + r = [b + \beta(S + x)] / (1 + \beta)$ .

The necessary conditions include:

$$(2.9) \quad \frac{\partial \mathcal{L}}{\partial x} = \frac{\beta}{1 + \beta} p'(Q)x + p(Q) - \lambda_X + \alpha \lambda_S + \mu_x = 0,$$

$$(2.10) \quad \dot{\lambda}_X = \delta \lambda_X - \frac{\partial \mathcal{L}}{\partial X} = \delta \lambda_X - \mu_X,$$

$$(2.11) \quad \dot{\lambda}_S = \delta \lambda_S - \frac{\partial \mathcal{L}}{\partial S} = \delta \lambda_S - \mu_S - \frac{\beta}{1 + \beta} p'(Q)x,$$

$$(2.12) \quad x \geq 0, \mu_x \geq 0, \mu_x x = 0,$$

$$(2.13) \quad X \geq 0, \mu_X \geq 0, \mu_X X = 0,$$

$$(2.14) \quad S \geq 0, \mu_S \geq 0, \mu_S S = 0,$$

and two transversality conditions:

$$(2.15) \quad \lim_{t \rightarrow +\infty} e^{-\delta t} \lambda_X(t) X(t) = 0,$$

$$(2.16) \quad \lim_{t \rightarrow +\infty} e^{-\delta t} \lambda_S(t) S(t) = 0,$$

and  $S(0) \geq 0$  and  $X^0 > 0$  given.

One can show that the solution of the problem is such that  $T^* < +\infty$ . In the rest of the chapter, we focus on this situation. Let  $p^*(t)$  be the optimal price path.

#### 2.4. Optimal Extraction Path and Price Dynamics

The optimal extraction path is summarized through the upcoming proposition.

**Proposition 4.** [*Optimal Path*]: *The optimal extraction path is such that the extracted quantity decreases while the recycled material quantity increases through time:*

$$\dot{x}^*(t) \leq 0 \text{ and } \dot{r}^*(t) \geq 0.$$

Proposition 4 states that the optimal level of extraction decreases through time. This result is in line with the standard Hotelling model: since the extractor discounts time, he chooses to extract larger quantities of resource today and less tomorrow. The



quantity of recycled material increases through time. This is a quite intuitive result. Indeed, extracted quantities become scrap. Hence, the stock of scrap increases and then the unit cost of recycling decreases. At the same time, extracted quantities decrease, which increases the market price and encourages recycling.

The following proposition summarizes the dynamic of the price of the resource.

**Proposition 5.** [*Price Dynamics*]: *The price of the final good is never decreasing if and only if the recyclability rate is sufficiently low. Formally, there exists  $\hat{\alpha} > 0$  such that  $\partial p^*/\partial t \geq 0$  for all  $t$  if and only if  $\alpha \leq \hat{\alpha}$ .*

Through the next corollary, we observe that the price of the resource is not monotonic.

**Corollary 6.** [*Non Monotonic Price*]: *If the recyclability rate is sufficiently large compared to the discount rate and the initial stock of the resource is sufficiently large, then the price of the final good first decreases and then increases. Formally, if  $\alpha > \hat{\alpha}$  there exists  $0 < \hat{t} < T^*$  such that  $\partial p^*/\partial t < 0$  if  $t \in [0, \hat{t})$  and  $\partial p^*/\partial t \geq 0$  if  $t \in [\hat{t}, T^*]$ .*

Proposition 5 and Corollary 6 can be illustrated thanks to the following numerical example:

**Numerical Example a:** Let  $X^0 = 1$ ,  $\beta = 1$  and  $\delta = 0.02$ . Figure 2.1 shows the optimal extraction path for different values of  $\alpha$ .

This figure shows that, in the case where the recyclability rate is high, the Hotelling's rule which states that the price of exhaustible resources increases over time does not hold. This behavior of the price is more consistent with what happens in many markets of exhaustible resources where the price usually follows cyclical phases.

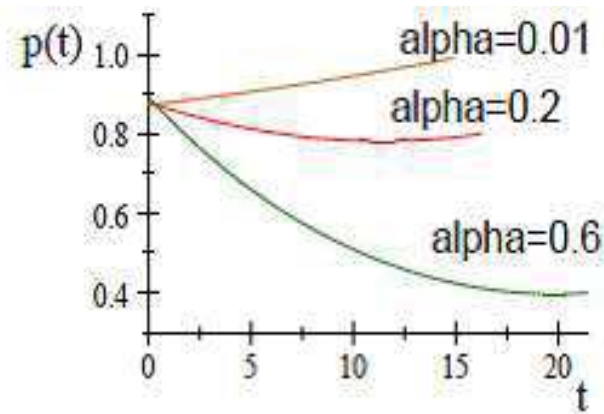


Figure 2.1. Non monotonic price dynamics ( $\alpha = 0.01; 0.2; 0.6$ )

## 2.5. The Role of Recyclability

**Proposition 7. [Exhaustion date and Recyclability]:** *The optimal exhaustion date increases with the initial stock and the recyclability rate of the resource. Formally:*

$$\frac{\partial T^*}{\partial X^0} > 0 \text{ and } \frac{\partial T^*}{\partial \alpha} > 0$$

Proposition 7 states that the date of exhaustion of the resource increases with the initial stock, which is intuitive. It also states that the exhaustion date increases with the recyclability rate of the resource. This suggests that recyclability delays extraction.

**Proposition 8. [Extraction and Recyclability]:** *Early extraction decreases while latter extraction increases with recyclability. Formally, there exists a date  $0 < \tilde{t} < T^*$  such that*

$$\frac{\partial x^*}{\partial \alpha} < 0 \text{ for } t \in [0, \tilde{t}) \text{ and } \frac{\partial x^*}{\partial \alpha} \geq 0 \text{ for } t \in (\tilde{t}, T^*).$$

Proposition 8 states that when the level of recyclability of the resource increases, then extraction is delayed. It first decreases and increases latter. The intuition of this result is as follows. When the resource is not exhausted ( $X > 0$ ), the dynamic of its

shadow price follows the Hotelling's rule, the (shadow) price of the resource grows at a rate equal to the discount rate,  $\dot{\lambda}_X/\lambda_X = \delta > 0$ . This means that the extracting firm has incentives to extract the resource early. However, unlike in a standard Hotelling model, the extractor faces the recycling fringe sector. When there is a stock of scrap ( $S > 0$ ) the dynamic of the shadow price of scrap is  $\dot{\lambda}_S = \delta\lambda_S + \frac{\beta}{1+\beta}x$ . If there is no extraction ( $x = 0$ ), then  $\dot{\lambda}_S/\lambda_S = \delta > 0$ , and then the owner of the resource has an incentive to delay extraction in order to maintain a small scrap stock (as long as  $\alpha > 0$ ). If the level of extraction is positive,  $\frac{\beta}{1+\beta}x > 0$ , which leads to a tendency for the shadow price of scrap to increase, which reinforces the incentives to delay extraction, as long as  $\alpha > 0$ . Hence, the higher the recyclability rate of the final good, the larger the resource owner incentives to postpone extraction. Since the resource is exhausted in finite time and the initial stock is fixed, extraction increases with the recyclability rate at some point in time.

**Numerical Example b:** Let  $X^0 = 1$ ,  $\beta = 1$  and  $\delta = 0.02$ . Figure 2.2 shows the optimal extraction path for different values of  $\alpha$ .

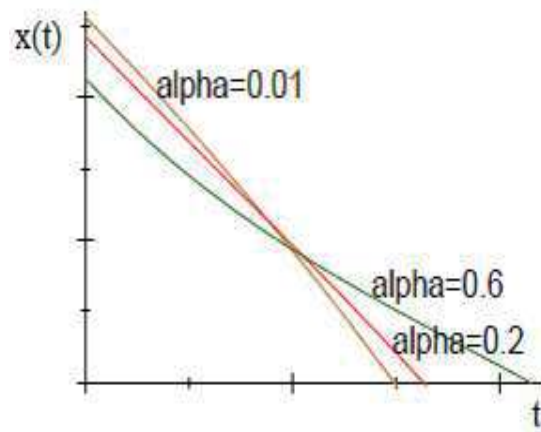


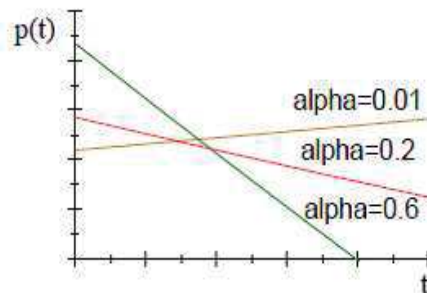
Figure 2.2. Extraction path ( $\alpha = 0.01; 0.2; 0.6$ )

**Proposition 9. [Price and Recyclability]:** *The market price first increases and then decreases with the recyclability rate. Formally, there exists a date  $0 < t' < T^*$  such that*

$$\frac{\partial p^*}{\partial \alpha} > 0 \text{ for } t \in [0, t') \text{ and } \frac{\partial p^*}{\partial \alpha} \leq 0 \text{ for } t \in (t', T^*).$$

This result shows that a higher recyclability rate of the resource is not always beneficial to consumers. In the short term, consumers are worse off while they are better off when the resource is sufficiently depleted. The intuition of this result is as follows. The price of the final good negatively depends on the extraction level and the stock of scrap. When the recyclability rate increases, the stock of scrap tends to increase. However, it increases with extraction with a factor  $\alpha < 1$ , thus the increase in recycling cannot compensate for the short run decrease in extraction. This explains why the price of the final good increases in the short run. When going close to the exhaustion date, an increase in the recyclability rate increases both the scrap stock and the level of extraction. This explains why the price of the final good decreases when going close to the exhaustion date. This result can be illustrated by the following numerical example.

**Numerical Example c:** Let  $X^0 = 1$ ,  $\beta = 1$  and  $\delta = 0.02$ . Figure 2.3 shows the optimal price path for different values of  $\alpha$ .



Recyclability and price dynamics ( $\alpha = 0.01; 0.2; 0.6$ )

## 2.6. Conclusion

Given the importance of phosphorus in the food production and due the anticipated depletion of the resource, recycling becomes a major challenge for many countries. In order to reduce their dependence on phosphate imports, many countries of European Union (like Germany, Netherlands, Sweden, etc.) have built recycling factories. Recycling is also undertaken in United States of America, in Canada, in China, in Japan. Using an optimal control model, we have shown that recycling may help to prolong the lifetime of phosphorus. However, recycling may decrease the short-run extraction, resulting then in a decline of consumers' surplus. The decrease of the price, obtained in our model in the case where the recyclability rate is high, contradicts Hotelling reasoning, which states that the price of exhaustible resources will increase over time. Indeed, in contrast to Hotelling (1931), the dynamic of the price may follow a downward phase in the short-run because of recycling. After reaching its minimum, the price follows an upward phase in the long-run. We have also shown other interesting results. Recycling postpones the exhaustion of phosphorus. Also, we have shown that consumers' surplus decreases with recyclability in the short-run. An implication of this result is that consumers may oppose the implementation of the recycling activities? At the first glance, one may reply by the positive. However, may be for other reasons related to a willing of conserving this precious resource or to the altruism, they can accept the implementation of the recycling activities, in spite the fact that they are worse off in terms of surplus. Among other results established in this chapter, we can stress the fact that higher is the recycling rate or higher is the level of the stock of phosphorus, later is the resource depleted. Such a result shows that, the announced dates of depletion of phosphorus can be modified by recycling and by new discoveries which increase phosphate reserves. We also show that higher is the discount rate, higher is the rate of extraction.

In this chapter (as in chapter 1), we have implicitly assumed that extracted phosphorus and recycled phosphorus are strategic substitutes. The next step would consist of investigating whether the established results will continue to hold in the case where both products exhibit strategic complementarity.

## Appendix

**Proof of Proposition 4:** Using the equilibrium recycling condition (2.6) and substituting, we have  $p(Q) = \frac{\beta}{1+\beta} (a - x - S)$ , where  $a = (1 - b + \beta) / \beta$ . The maximization problem then formally writes:

$$(2.17) \quad \underset{\{x\}}{Max} \int_0^{+\infty} \frac{\beta e^{-\delta t}}{1+\beta} (a - x - S) x dt,$$

subject to (2.6), (2.1), (2.2),  $X, S, x \geq 0$ ,  $X^0$  and  $S^0$  given.

Thus, for the new problem, the necessary conditions include:

$$(2.18) \quad \frac{\partial \mathcal{L}}{\partial x} = a - 2x - S - \lambda_X + \alpha \lambda_S + \mu_x = 0,$$

$$(2.19) \quad \dot{\lambda}_X = \delta \lambda_X - \frac{\partial \mathcal{L}}{\partial X} = \delta \lambda_X - \mu_X,$$

$$(2.20) \quad \dot{\lambda}_S = \delta \lambda_S - \frac{\partial \mathcal{L}}{\partial S} = \delta \lambda_S - \mu_S + x,$$

$$(2.21) \quad x \geq 0, \mu_x \geq 0, \mu_x x = 0,$$

$$(2.22) \quad X \geq 0, \mu_X \geq 0, \mu_X X = 0,$$

$$(2.23) \quad S \geq 0, \mu_S \geq 0, \mu_S S = 0,$$

and two transversality conditions:

$$(2.24) \quad \lim_{t \rightarrow +\infty} e^{-\delta t} \lambda_X(t) X(t) = 0,$$

$$(2.25) \quad \lim_{t \rightarrow +\infty} e^{-\delta t} \lambda_S(t) S(t) = 0.$$

Let us assume that the solution is such that  $x(t) > 0$  and  $X(t) > 0$  over  $[0, T]$  and  $x(t) = X(t) = 0$  for  $t \geq T$ . We also assume that  $S(t) > 0$  for all  $t > 0$ .

**First consider the first phase where  $t \in [0, T]$ .** Since  $x(t) > 0$ ,  $X(t) > 0$  and  $S(t) > 0$ , using (2.21), (2.22), and (2.23), we have  $\mu_x = \mu_X = \mu_S = 0$ . Then (2.19) writes

$$(2.26) \quad \dot{\lambda}_X = \delta \lambda_X,$$

and then

$$(2.27) \quad \lambda_X = c_1 e^{\delta t},$$

where  $c_1$  is a constant to be determined latter.

Conditions (2.18), and (2.20) write

$$(2.28) \quad a - 2x - S - c_1 e^{\delta t} + \alpha \lambda_S = 0,$$

and,

$$(2.29) \quad \dot{\lambda}_S = \delta \lambda_S + x,$$

Differentiating (2.28) with respect to time, we find

$$(2.30) \quad -2\dot{x} - \dot{S} - \delta c_1 e^{\delta t} + \alpha \dot{\lambda}_S = 0.$$

Using (2.28) and (2.30), we find

$$(2.31) \quad -2\dot{x} - \dot{S} - \delta c_1 e^{\delta t} - \delta (a - 2x - S - c_1 e^{\delta t}) + \alpha (\dot{\lambda}_S - \delta \lambda_S) = 0,$$



or,

$$(2.32) \quad -2\dot{x} - \dot{S} + \delta S + (\alpha + 2\delta)x - \delta a = 0,$$

Differentiating (2.2) with respect to time, we obtain

$$(2.33) \quad \ddot{S} = \alpha\dot{x}.$$

Substituting (2.33) and (2.2) into (2.32), and rearranging, we have

$$(2.34) \quad \ddot{S} - \delta\dot{S} - \frac{\alpha\delta}{2}S + \frac{\alpha\delta}{2}a = 0.$$

Solving for the stock of scrap  $S$  we find

$$(2.35) \quad S = c_2 e^{\gamma^+ t} + c_3 e^{\gamma^- t} + a,$$

where  $\gamma^+ = \frac{\delta + \sqrt{\delta(2\alpha + \delta)}}{2}$  and  $\gamma^- = \frac{\delta - \sqrt{\delta(2\alpha + \delta)}}{2}$ .

Differentiating (2.35) with respect to time, we obtain

$$(2.36) \quad \dot{S} = \gamma^+ c_2 e^{\gamma^+ t} + \gamma^- c_3 e^{\gamma^- t}.$$

Using (2.2), we have

$$(2.37) \quad x = \frac{\gamma^+}{\alpha} c_2 e^{\gamma^+ t} + \frac{\gamma^-}{\alpha} c_3 e^{\gamma^- t}.$$

Substituting (2.37) into (2.29), we obtain

$$(2.38) \quad \dot{\lambda}_S - \delta\lambda_S = \frac{\gamma^+}{\alpha} c_2 e^{\gamma^+ t} + \frac{\gamma^-}{\alpha} c_3 e^{\gamma^- t}.$$

Solving for the shadow price of the stock of scrap  $\lambda_S$  we find

$$(2.39) \quad \lambda_S = De^{\delta t} + \frac{\gamma^+}{\alpha(\gamma^+ - \delta)} c_2 e^{\gamma^+ t} - \frac{\gamma^-}{\alpha(\delta - \gamma^-)} c_3 e^{\gamma^- t}.$$

Using  $X^0 - X(t) = \int_0^t x dt$  and integrating (2.37) between 0 and  $t$ , we find

$$(2.40) \quad X^0 - X(t) = \frac{1}{\alpha} \left( c_2 \left( e^{\gamma^+ t} - 1 \right) + c_3 \left( e^{\gamma^- t} - 1 \right) \right).$$

**Now consider the second phase where  $t \geq T$ .** We have  $x(t) = 0 = X(t) > 0$  and  $S(t) > 0$ . Using (2.23), we have  $\mu_S = 0$ . Condition (2.20) writes

$$(2.41) \quad \dot{\lambda}_S = \delta \lambda_S,$$

and then

$$(2.42) \quad \lambda_S = c_5 e^{\delta t},$$

where  $c_5$  is a constant to be determined latter.

Notice that  $\dot{S} = \alpha x = 0$ , and then

$$(2.43) \quad S = c_4,$$

where  $c_4$  is a constant to be determined latter.

Using (2.43) and (2.42), transversality constraint (2.25) becomes

$$(2.44) \quad c_4 c_5 = 0.$$

Assume  $c_5 \neq 0$ . Then, using (2.43) at  $t = T$ , we have  $S(T) = c_4 = 0$ . Combining (2.35) and (2.40) and taking  $t = T$ , we have  $\alpha X^0 = S(T)$ . Hence, we must have  $X^0 = 0$ , which

is false. We conclude that  $c_5 = 0$  and then, for  $t \geq T$ , we have

$$(2.45) \quad \lambda_S = 0.$$

In order to solve for  $c_1, c_2, c_3, c_4, D$  and  $T$ , let us to focus on solutions such that  $\lambda_S$  is continuous. Using (2.39) and (2.45) at  $t = T$ , we obtain

$$(2.46) \quad De^{\delta T} + \frac{\gamma^+}{\alpha(\gamma^+ - \delta)} c_2 e^{\gamma^+ T} - \frac{\gamma^-}{\alpha(\delta - \gamma^-)} c_3 e^{\gamma^- T} = 0.$$

Using  $x(T) = 0$  and (2.37), we have

$$(2.47) \quad \gamma^+ c_2 e^{\gamma^+ T} + \gamma^- c_3 e^{\gamma^- T} = 0.$$

Using  $X(T) = 0$  and (2.40), we have

$$(2.48) \quad \alpha X^0 = c_2 \left( e^{\gamma^+ T} - 1 \right) + c_3 \left( e^{\gamma^- T} - 1 \right).$$

Using (2.35) at  $t = 0$ , we have

$$(2.49) \quad S^0 = c_2 + c_3 + a.$$

Using (2.28) and (2.35) at  $t = T$ , we obtain

$$(2.50) \quad c_2 e^{\gamma^+ T} + c_3 e^{\gamma^- T} + c_1 e^{\delta T} = 0.$$

Using (2.35) and (2.43) at  $t = T$ , we have

$$(2.51) \quad c_4 = c_2 e^{\gamma^+ T} + c_3 e^{\gamma^- T} + a.$$

Solving for  $c_1, c_2, c_3, c_4, D$  and  $T$  thanks to conditions (2.46)-(2.51), we obtain

$$\begin{aligned}
c_1 &= \frac{\gamma^+ - \gamma^-}{\gamma^+ e^{\gamma^+ T} - \gamma^- e^{\gamma^- T}} (a - S^0), \\
c_2 &= -\frac{\gamma^- e^{\gamma^- T}}{\gamma^+ e^{\gamma^+ T} - \gamma^- e^{\gamma^- T}} (S^0 - a), \\
c_3 &= \frac{\gamma^+ e^{\gamma^+ T}}{\gamma^+ e^{\gamma^+ T} - \gamma^- e^{\gamma^- T}} (S^0 - a), \\
c_4 &= (S^0 - a) \frac{(\gamma^+ - \gamma^-) e^{\delta T}}{\gamma^+ e^{\gamma^+ T} - \gamma^- e^{\gamma^- T}} + a, \\
D &= \frac{a - S^0}{\alpha} \frac{\gamma^+ - \gamma^-}{\gamma^+ e^{\gamma^+ T} - \gamma^- e^{\gamma^- T}},
\end{aligned}$$

and the exhaustion date  $T^*$  is implicitly characterized by :

$$(2.52) \quad \alpha X^0 = (a - S^0) \left( 1 - \frac{\gamma^+ - \gamma^-}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}} e^{\delta T^*} \right).$$

We conclude that the optimal extraction path is, for  $t \in [0, T]$  :

$$(2.53) \quad x^*(t) = \frac{(a - S^0) \delta}{2} \left( \frac{e^{\gamma^+ T^*} e^{\gamma^- t} - e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}} \right),$$

the stock of scrap is, for  $t \in [0, T]$ ,

$$(2.54) \quad S^*(t) = (a - S^0) \left( 1 - \frac{\gamma^+ e^{\gamma^+ T^*} e^{\gamma^- t} - \gamma^- e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}} \right),$$

and the market price, for  $t \in [0, T]$ ,

$$(2.55) \quad p^*(t) = S^0 + (a - S^0) \left( \frac{\gamma^+ - \gamma^-}{2} \right) \frac{e^{\gamma^+ T^*} e^{\gamma^- t} + e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}.$$

Since  $\gamma^+ > 0 > \gamma^-$ , the extraction level  $x^*(t)$  characterized in (2.53) decreases through time, while the stock of scrap, since increases through time,  $\dot{S}^*(t) = \alpha x^*(t) \geq 0$ .

Recycling is given by

$$(2.56) \quad r^*(t) = \frac{b}{\beta} + a \left( 1 - \frac{\left(\gamma^+ + \frac{\delta}{2\beta}\right) e^{\gamma^+ T^*} e^{\gamma^- t} - \left(\gamma^- + \frac{\delta}{2\beta}\right) e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}} \right),$$

and it is increasing through time.  $\square$

**Proof of Proposition 5:** From (2.55) we know that the price is

$$(2.57) \quad p^*(t, \alpha) = \frac{a}{2} \sqrt{\delta(2\alpha + \delta)} \frac{e^{\gamma^+ T^*} e^{\gamma^- t} + e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}},$$

The sign of the derivative with respect to time is given by

$$(2.58) \quad \frac{\partial p^*}{\partial t} \propto \gamma^- e^{\gamma^+ T^*} e^{\gamma^- t} + \gamma^+ e^{\gamma^- T^*} e^{\gamma^+ t},$$

which is positive if and only if

$$(2.59) \quad t \geq T^* + \frac{1}{\gamma^+ - \gamma^-} \ln \left( 1 - \frac{\delta}{\gamma^+} \right).$$

Hence,  $\frac{\partial p^*}{\partial t} \geq 0$  for all  $t \in [0, T^*]$  if and only if

$$(2.60) \quad T^* \leq \frac{1}{\gamma^+ - \gamma^-} \ln \left( \frac{\gamma^+}{\gamma^+ - \delta} \right).$$

We know from Proposition 7 that the left hand side of condition (2.60) is increasing with  $\alpha$ . The derivative of the right hand side with respect to  $\gamma^+$  is  $\frac{-1}{(2\gamma^+ - \delta)^2} \left( 2 \ln \frac{\gamma^+}{\gamma^+ - \delta} + \frac{\delta(2\gamma^+ - \delta)}{\gamma^+(\gamma^+ - \delta)} \right) < 0$ , thus it is decreasing with  $\alpha$ . When  $\alpha$  goes to 0,  $\gamma^+$  goes to  $\delta$  and then the right hand side in (2.60) goes to  $+\infty$ . A first order approximation of (2.52) at  $\alpha = 0$  leads to  $\left( \frac{X^0}{a} \delta + 1 - \delta T^* \right) e^{\delta T^*} \simeq 1$  and the solution of this equation is  $T^* < +\infty$  because the left hand side is  $\frac{X^0}{a} \delta + 1 > 1$  at  $T^* = 0$ , it increases up to  $T^* = \frac{X^0}{a}$  and then decreases and goes to  $-\infty$  when  $T^* \rightarrow +\infty$ . This concludes the proof.  $\square$

**Proof of Corollary 6:** The result directly follows from the proof of Proposition 5.  $\square$

**Proof of Proposition 7:** The optimal exhaustion date is implicitly characterized by

(2.52), which can be rewritten as:

$$(2.61) \quad f(\gamma^+, T^*, \alpha, X^0, \delta) \equiv 1 - \frac{2\gamma^+ - \delta}{\gamma^+ e^{\gamma^+ T^*} + (\gamma^+ - \delta) e^{(\delta - \gamma^+) T^*}} e^{\delta T^*} - \frac{\alpha X^0}{a} = 0,$$

where  $\gamma^+ = (\delta + \sqrt{\delta(2\alpha + \delta)})/2$ . Its derivative with respect to  $T^*$  is given by

$$(2.62) \quad \frac{\partial f}{\partial T^*} = \frac{\gamma^+ (\gamma^+ - \delta) (2\gamma^+ - \delta) e^{\delta T^*}}{(\gamma^+ e^{\gamma^+ T^*} + (\gamma^+ - \delta) e^{(\delta - \gamma^+) T^*})^2} (e^{\gamma^+ T^*} - e^{-(\gamma^+ - \delta) T^*}).$$

Since  $\gamma^+ \geq \delta$ , we have

$$(2.63) \quad \frac{\partial f}{\partial T^*} > 0.$$

The derivative of  $f$  with respect to  $\gamma^+$  is given by

$$(2.64) \quad \frac{\partial f}{\partial \gamma^+} = - \frac{\delta (e^{\gamma^+ T^*} - e^{-(\gamma^+ - \delta) T^*}) + (\gamma^+ e^{\gamma^+ T^*} - (\gamma^+ - \delta) e^{-(\gamma^+ - \delta) T^*}) (\gamma^+ - \delta) T^*}{(\gamma^+ - \delta)^2 \left( \frac{\gamma^+}{\gamma^+ - \delta} e^{\gamma^+ T^*} + e^{-(\gamma^+ - \delta) T^*} \right)^2} e^{\delta T^*}.$$

Since  $\gamma^+ > \delta$ , we have

$$(2.65) \quad \frac{\partial f}{\partial \gamma^+} < 0.$$

The derivative of  $f$  with respect to  $\alpha$  is

$$(2.66) \quad \frac{\partial f}{\partial \alpha} = -X^0/a < 0$$

Using (2.61) and the implicit function theorem, we have:

$$(2.67) \quad \frac{\partial T^*}{\partial X^0} = - \frac{\partial f / \partial X^0}{\partial f / \partial T} = \frac{\alpha/a}{\partial f / \partial T} > 0.$$

Using the implicit function theorem again, and  $\partial\gamma^+/\partial\alpha > 0$ , (2.65), (2.63) and (2.66), the derivative of the exhaustion date with respect to the recyclability rate is such that:

$$(2.68) \quad \frac{\partial T^*}{\partial \alpha} = -\frac{(\partial f/\partial \gamma^+)(\partial \gamma^+/\partial \alpha) + \partial f/\partial \alpha}{\partial f/\partial T} > 0.$$

**Proof of Proposition 8:** The proof proceeds in three steps. We first show that the growth rate of extraction is decreasing through time. Second, we show that the growth rate is increasing with the recyclability rate. We then combine these properties in order to prove the result.

Differentiating (2.53), we can write the growth rate of extraction:

$$(2.69) \quad \tau(\gamma^+, T^*) \equiv \frac{\dot{x}^*(t)}{x^*(t)} = -\frac{(\gamma^+ - \delta)e^{(2\gamma^+ - \delta)(T^* - t)} + \gamma^+}{e^{(2\gamma^+ - \delta)(T^* - t)} - 1}.$$

The derivative of the growth rate with respect to  $T^*$  is

$$(2.70) \quad \frac{\partial \tau}{\partial T^*} = \frac{\gamma^+ + \delta + (2\gamma^+ - \delta)\gamma^+}{(e^{(2\gamma^+ - \delta)(T^* - t)} - 1)^2} \delta e^{(2\gamma^+ - \delta)(T^* - t)} > 0.$$

The derivative of the growth rate with respect to  $\gamma^+$  is

$$\frac{\partial \tau}{\partial \gamma^+} = \delta \frac{G(t)}{(e^{(2\gamma^+ - \delta)(T^* - t)} - 1)^2},$$

where  $G(t) = 2(T^* - t)(2\gamma^+ - \delta)e^{(2\gamma^+ - \delta)(T^* - t)} + 1 - e^{2(2\gamma^+ - \delta)(T^* - t)}$ . Notice that  $G'(t) = -2(T^* - t)(2\gamma^+ - \delta)^2 e^{(2\gamma^+ - \delta)(T^* - t)} < 0$  and  $G(T^*) = 0$ . Hence  $G(t) > 0$  and then

$$(2.71) \quad \frac{\partial \tau}{\partial \gamma^+} > 0.$$

We know from Proposition 7 that  $T^*$  increases with  $\alpha$  and we also know that  $\gamma^+$  increases with  $\alpha$ . Using (2.70) and (2.71), we conclude that

$$(2.72) \quad \frac{d\tau}{d\alpha} > 0.$$

In other words, we have

$$(2.73) \quad \frac{\partial^2 \ln(x)}{\partial t \partial \alpha} > 0.$$

Hence  $\ln x$  has the single crossing property with respect to time and the recyclability rate.

According to Proposition 7, the exhaustion date increases with recyclability,  $\frac{\partial T^*}{\partial \alpha} > 0$ . Hence recyclability necessarily increases extraction when time gets close to the exhaustion date. Since the initial stock does not depend on the recyclability rate, recyclability necessarily decreases extraction at some point in time. Thanks to the single-crossing property, there exists a date  $0 < \tilde{t} < T^*$  such that  $\frac{\partial x^*}{\partial \alpha} < 0 \iff t < \tilde{t}$ .  $\square$

**Proof of Proposition 9:**

The stock of scrap can be rewritten as follows:

$$S^*(t) = F(\gamma^+, T^*) = a \left( 1 - \frac{\gamma^+ e^{\gamma^+(T^*-t)} + (\gamma^+ - \delta) e^{-(\gamma^+ - \delta)(T^*-t)}}{\gamma^+ e^{\gamma^+ T^*} + (\gamma^+ - \delta) e^{-(\gamma^+ - \delta) T^*}} e^{\delta t} \right).$$

The derivative of this function with respect to  $T^*$  is:

$$(2.74) \quad \frac{\partial F}{\partial T^*} = -a \frac{\gamma^+ (\gamma^+ - \delta) (2\gamma^+ - \delta) \left[ e^{\delta T - \gamma^+ t} - e^{\delta T + (\gamma^+ - \delta)t} \right]}{(\gamma^+ e^{\gamma^+ T^*} + (\gamma^+ - \delta) e^{-(\gamma^+ - \delta) T^*})^2} e^{\delta t} \geq 0,$$

and its derivative with respect to  $\gamma^+$  is

$$(2.75) \quad \frac{\partial F}{\partial \gamma^+} = -a \frac{-t (\gamma^+)^2 e^{\gamma^+(2T^*-t)} + \left[ (2T^* - t) (\gamma^+)^2 - \delta \right] e^{\delta T - \gamma^+ t} - \left[ 2T^* (\gamma^+)^2 - \delta \right] e^{\delta T + (\gamma^+ - \delta)t}}{(\gamma^+ e^{\gamma^+ T^*} + (\gamma^+ - \delta) e^{-(\gamma^+ - \delta) T^*})^2} e^{-\delta t} \geq 0.$$



Since  $T^*$  and  $\gamma^+$  both increase with  $\alpha$ , using (2.74) and (2.75) we conclude that  $S^*$  increases when  $\alpha$  increases.

Now consider the growth rate of the price. Using (2.55), it can be written as follows:

$$(2.76) \quad \frac{\dot{p}^*}{p^*} = \frac{\gamma^+ - (\gamma^+ - \delta) e^{(2\gamma^+ - \delta)(T^* - t)}}{1 + e^{(2\gamma^+ - \delta)(T^* - t)}} \equiv H(\gamma^+, T^*).$$

Its derivative with respect to  $\gamma^+$  is given by:

$$(2.77) \quad \frac{\partial H}{\partial \gamma^+} = \frac{1 - e^{2(2\gamma^+ - \delta)(T^* - t)} - 2(2\gamma^+ - \delta)(T^* - t)e^{(2\gamma^+ - \delta)(T^* - t)}}{(1 + e^{(2\gamma^+ - \delta)(T^* - t)})^2} \leq 0.$$

Since  $H$  is also decreasing with  $T^*$  and both  $T^*$  and  $\gamma^+$  both increase with  $\alpha$ , we conclude that

$$(2.78) \quad \frac{\partial^2 \ln p^*}{\partial t \partial \alpha} < 0.$$

This means that  $\ln p^*$  has the single-crossing property with respect to  $t$  and  $\alpha$ . We know that

$$\begin{aligned} p^*(T^*) &= 1 - S^*(T^*) - x^*(T^*) \\ &= 1 - \alpha X^0. \end{aligned}$$

Then  $p^*(T^*)$  decreases with  $\alpha$ . Moreover, we have

$$(2.79) \quad \begin{aligned} p^*(0) &= 1 - S^*(0) - x^*(0) \\ &= 1 - x^*(0). \end{aligned}$$

Using Proposition 8, we know that  $x^*$  decreases with  $\alpha$  at  $t = 0$ . Hence  $p^*$  increases with  $\alpha$  at  $t = 0$ . Using the single-crossing property (2.78), we conclude that there exists  $t' \in (0, T^*)$  such that  $\partial p^* / \partial \alpha > 0 \iff t' \in [0, T^*)$ .  $\square$

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## CHAPTER 3

## The Influence of Recycling on Monopoly Production under Capacity Constraints

### Abstract

We consider a monopolistic market in which the amount of virgin product available over two consecutive periods is limited. In the second-period, the monopoly competes with a recycling sector whose production capacity depends on the monopolist's first-period production. We consider the possibility that extracted quantities and recycled quantities are strategic substitutes or strategic complements. We show that recycling has two effects on prior price of raw products: a "recycling capacity" effect and a strategic effect. The recycling capacity effect always increases prior price of raw material, i.e. the monopoly decreases its first-period production in order to limit recycling quantities. The strategic effect increases the prior price of raw materials (decreases their prior production) only raw and recycled products are strategic complements, whereas it decreases prior price (increases the prior production) if they are strategic substitutes. We then use an illustrative example to show that both effects may dominate and that the first-period production increases or decreases accordingly.

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**Keywords:** Recycling, Strategic substitutability, Strategic Complementarity, Capacity Constraints.

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### 3.1. Introduction

Due to increased interest in resource conservation<sup>1</sup> and in waste handling<sup>2</sup>, the recycling of raw materials has grown in significance in most industrialized economies (Martin, 1982; André and Cerdà, 2003). The issue of recycling has been much more explored in the field of aluminum via the case of Alcoa. Several authors have explored whether the market power of Alcoa was affected by the presence of a competitive recycling sector or not. This earlier literature has implicitly assumed that the raw and the recycled products are substitutes. However, there is no evidence. The consideration of strategic complementarity, in the present chapter, generates a number of new conclusions, as regards the following questions. What is the optimal path of extraction for a capacity-constrained monopolist that faces a competitive recycling sector? Do the earlier established results still hold? It is noteworthy to highlight that the capacity constraint means, here, the resource is exhausted at the end of the second-period.

The purpose of this chapter is to analyze the effect of recycling on the second-period marginal revenue of a capacity-constrained monopolist and on its first-period extraction under the assumption that the recycled and the raw products may be either strategic substitutes or strategic complements. Toward this goal, we use a Stackelberg two-period model where two firms compete in the second-period. The recycler enters the market in the second-period.

Our main results are as follows. The effect of recycling on the second-period marginal revenue of the monopolist and on its first-period production depends on whether the recycled and the raw products are strategic complements or strategic substitutes.

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<sup>1</sup>Weinstein and Zeckhauser (1974), Huhtala (1994), Cordell and al. (2009), Weikard and Seyhan (2009) indicate that the use of recycled materials enables scarce resources to be saved.

<sup>2</sup>Lund (1990) and Sigman (1995) argue that recycling is an environmental friend.

In the case where they are strategic complements, we show that the second-period marginal revenue of the monopolist increases in recycling, whereas its first-period production decreases in recycling. Conversely, in the case where both products are strategic substitutes, the effect of recycling depends on whether the recycling capacity effect outweighs the strategic effect and, vice versa. Indeed, if the recycling capacity effect dominates the strategic effect, recycling increases the second-period marginal revenue of the monopolist and decreases its first-period extraction, while the opposing results are obtained if the strategic effect is higher than the recycling capacity effect.

This chapter is based on several strands of the literature: the first one is the literature that focused on the case of Alcoa, the second one relates to the "green paradox", and the third one refers to the industrial economics literature on multi-period models of capacity constraints. With respect to the first strand, Gaskins (1974) shows that the existence of a competitive recycling sector leads to an increase in the price set by Alcoa compared to the monopoly price, in the short-run. This means that Alcoa's production decreases in the short term. Our linear model yields the opposite result under the assumption that the resource is exhausted over the two periods. Our two-period model shows that Gaskins's conclusions are reversed in the case where the exhaustible resource and the substitute are strategic substitutes and the strategic effect outweighs the recycling capacity effect. In such a situation, the second-period marginal revenue of the monopolist decreases, resulting in the decline of its second-period production which, in turn, triggers the rise of its first-period production. Swan (1980), Martin (1982) and Grant (1999) conclude that Alcoa's market power is not affected by the presence of recycling. In other words, the long run price charged by Alcoa (which corresponds to the price set by the monopolist in the second-period, within our framework) does not fall. Our model shows the contrary when the exhaustible resource and the substitute are strategic substitutes.

The second strand of the literature relates to the "green paradox". Since Sinn (2008), future announced policies that increase present emissions are considered to exhibit a "green paradox"<sup>3</sup>. While Sinn was mainly concerned with the potentially harmful effects of increasing future carbon taxes, several authors noted that a similar effect can be caused by the development of a clean backstop<sup>4</sup> technology (Michielsen, 2011). The sensitivity of the green paradox to the presence of a clean backstop, i.e. a clean substitute<sup>5</sup>, is explored by Ploeg and Withagen (2009), who find that the green paradox occurs for clean and expensive backstops. Michielsen (2011) shows that the future availability of a clean backstop increases initial emissions or the first-period extraction. Ploeg and Withagen (2012) focus on the case where marginal extraction costs of the exhaustible resource depend on the existing stock and assume that the substitute is unlimited. They find that the green paradox occurs in the situation where the cost of the backstop decreases, provided that the backstop remains expensive such that the stock of the non-renewable resource is eventually exhausted. Grafton and al. (2012) provide necessary and sufficient conditions for the green paradox to hold. They decompose the effect of a biofuel<sup>6</sup> subsidy into two effects: a direct effect<sup>7</sup> and an indirect effect of the subsidy. They find that a green paradox prevails only in the case where the indirect effect dominates the direct

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<sup>3</sup>Note that the term green paradox refers to the fact that the owners of the resource shift their supply intertemporally. The expression "green" makes reference to the green technologies (Gerlagh, 2009).

<sup>4</sup>In the present chapter, the clean backstop technology corresponds to the recycling technology. Beir and Girmens (2009) highlight that there is plenty of evidence to justify the difference in pollution levels between production of raw products and production of recycled goods. Firstly, recycling reduces the production of waste. Secondly, recycling avoids the pollution related to mining. Thirdly, it avoids the pollution caused by the primary production. In particular, the latter produces more greenhouse gases than secondary production, essentially because of the energy resources required to produce primary aluminum.

<sup>5</sup>The substitute can be the cause of a green paradox due to a declining of its price, either because of increasing subsidies or technological improvement (Hoel, 2010). Hoel (2008) shows that carbon emissions may also increase as a consequence of an immediate and once and for all downward shift in the cost of producing a substitute.

<sup>6</sup>The biofuel is a substitute.

<sup>7</sup>In their analysis, the direct effect is taken as "pro-green", whereas the indirect effect is considered to be "anti-green".



effect. Gerlagh (2011) differentiates between a weak and a strong green paradox. He highlights that a weak green paradox holds when a cheaper clean energy technology increases current emissions, whereas a strong green paradox arises when the cheaper clean technology increases cumulative damages associated with emissions as well, evaluated at the net present value. In other words, the weak green paradox refers to an immediate effect, while the strong green paradox makes reference to an aggregate welfare effect. Assuming the presence of a perfect or an imperfect substitute and the existence of fossil fuel extraction costs, he finds that increasing the latter counteracts the strong green paradox in the case where the substitute is perfect, while with imperfect energy substitute both the weak and the strong green paradox may vanish. Hoel (2008) investigates the effect of an improvement of the technology for producing the substitute. Such improvement lowers the cost of production. He finds that the short run greenhouse gas emissions may increase as a response to the reduced cost of the substitute. The same issue is explored by Strand (2007) who argues that the improvement of the technology, which will make carbon redundant in the future may increase present emissions. The overall conclusion of this line of research is that the future presence of a clean substitute increases the first-period production.

The present chapter complements the existing literature by considering that the exhaustible resource (virgin resource) and the backstop (recycling) can exhibit either strategic substitutability or strategic complementarity. To the best of our knowledge, there is no paper which considers that both goods can exhibit strategic complementarity. This consideration is of significant importance in the sense that it modifies the previous results. We show that, under strategic substitutability, the traditional result, generally observed within the context of the green paradox, holds only under some specified conditions (for instance, in the case where the recycling capacity effect is lower than the

strategic effect). In this situation, the discounted second-period marginal revenue of the monopolist is decreased, resulting in the slowdown of its second-period production. Since the resource is depleted over the two periods, the latter induces the monopoly to enhance its first-period production. This result is reversed in the case where the recycling capacity effect is larger than the strategic effect. In such a situation, the discounted second-period marginal revenue of the monopolist is increased, resulting in the increase of its second-period production. As the resource constraint is binding, the latter induces the monopoly to decrease its first-period production. Moreover, if both the recycled and the raw products exhibit strategic complementarity, the traditional result obtained within the context of the green paradox is always reversed in the sense that the strategic effect and the recycling capacity effect work in the same direction. Since both effects are positive, the discounted second-period marginal revenue of the monopoly is increased, resulting in the increase of its second-period production. Accordingly, the monopoly reduces its first-period production.

The third strand of the literature refers to the multi-period industrial competition models with capacity constraints. Tsutsui (1996) considers a dynamic Cournot competition. He shows that the presence of the capacity constraints leads the firms to cut back, voluntarily, their current output<sup>8</sup>, in order to sustain higher future prices. In contrast, in the present chapter, when the recycled and the raw products exhibit strategic substitutability and the strategic effect larger than the recycling capacity effect, we show

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<sup>8</sup>The author explains the intuition behind this result as follows. In a static Cournot framework, the firms overproduce. In fact, the firms' inability to cooperate leads them to produce too much relative to joint-profit maximization output. However, because of the noninstantaneous price adjustment in this Cournot dynamic game, there also exists another (dynamic) incentive which mitigates the firms' overproduction incentive. Indeed, the firms take into account the effect of their output on the subsequent prices. That is, the firms recognize that lowering current output will sustain higher prices and boost future profits. Thus the dynamic incentive lets the firms reduce their current output. A capacity constraint limits the amount gains that each firm is able to realize from cheating and, at the same time, the constraint can reinforce the dynamic incentive.

that the current production increases. Martinez-de-Albéniz and Talluri (2011) investigate price competition for an oligopoly in a dynamic setting, where each of the sellers has a fixed number of units available for sale over a fixed number of periods. They assume that demand is stochastic and find that prices decrease in the first periods before increasing in the last periods. Gabszewicz and Poddar (1997) use a Cournot two-stage model where firms choose capacity in the first stage without knowing what will happen in the future, and output levels in the second stage, knowing which state is realized. They explore the effect of uncertain demand on firm's capacity decisions and highlight that firms are in excess capacity compared with the capacity they would choose in the Cournot certainty similar model. In the present chapter, the owner of the resource has an exogenous capacity constraint and recycling is limited by the amount of recyclable material.

The remainder of this chapter is organized as follows. Section 3.2 establishes the relationship between Cournot competition and the strategic complementarity. In section 3.3, we outline the model. The main conclusions and some further research lines are given in section 3.4 and all proofs are relegated to the appendix in section 3.5.

### 3.2. Cournot competition and strategic complementarity

There is a common understanding that quantity competition (Cournot competition) is related to strategic substitutability<sup>9</sup> and price competition (Bertrand competition) to strategic complementarity<sup>10</sup>. Bulow and al. (1985) argue that this does not always hold.

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<sup>9</sup>Note that two products are substitutes if a higher output for one reduces price for the other in Cournot competition (Amir, 2001). Strategic substitutability of the Cournot game holds if the cross partial derivative of any firm's profit function with respect to its own output and to the output of any other firm is negative.

<sup>10</sup>The actions of two players are said to be strategic complements if the marginal profitability of a player increases with the action of the rival (Vives, 1990). Strategic complementarity of the Cournot game holds if the cross partial derivative of any firm's profit function with respect to its own output and to the output of any other firm is nonnegative.

Bulow and al. (1985) argue that one cannot determine whether products are strategic substitutes or complements without empirically analyzing a market.

They stress that quantity competition and constant elasticity demand may yield strategic complements, but a linear demand curve with the same elasticity around equilibrium will always yield strategic substitutes. They highlight also that price competition can yield strategic substitutes. Amir and Jin (2001) stress that in an oligopoly with linear demand, quantity competition can exhibit strategic complementarity. With an analytical example (example 2), we illustrate how Cournot competition can, under some specified conditions, exhibit strategic complementarity.

### 3.3. A two-period Model

Consider an industry in which one firm has access to a finite amount of a product denoted  $S \geq 0$ . This provides the firm a monopoly position. The monopolist shares  $S$  between the two periods and sells  $q_t \geq 0$  units of the product at period  $t = 1, 2$ . The product can be recycled once it has been consumed, i.e. at time  $t = 2$ . The amount of recycled product is limited by the quantity of product produced and the recycling technology is such that a share  $\alpha \in (0, 1)$  can be recycled. In other words, when  $q_1 \geq 0$  units of the raw product are consumed in period 1, only  $r = \alpha q_1$  units of the recycled product can be produced in period 2. At time  $t = 1$ , the monopolist's profit is a concave function of the extracted quantity,  $\pi_1(q_1)$  with  $\pi_1'' \leq 0$ . At time  $t = 2$ , the monopolist is contested by a competitive recycling sector. As raw and recycled materials compete on the market, the profit of the monopolist is a decreasing function of the recycled quantity,  $\pi_2(q_2, r)$  with  $\frac{\partial \pi_2}{\partial r}(q_2, r) \leq 0$ . The second-period profit of the monopolist is discounted at a common discount factor  $\delta \in [0, 1]$ . We assume that the discounted profit,  $\pi_1 + \delta \pi_2$ , is strictly concave in  $(q_1, q_2)$ .

It is noteworthy that we do not assume that the profit function of the monopoly remains constant over time, thus our analysis encompasses cases in which demand and/or costs change from period 1 to period 2. The flexibility of the demand appears to be

important, since it seems likely that it changes over time<sup>11</sup>. We also do not make assumptions on the substitutability or complementarity of the raw and recycled products as regards consumers preferences. This is an innovative approach because the existing literature considers that recycling (or the backstop) and extraction (or the exhaustible resource) are substitutes (see Weikard and Seyhan, 2009; Gaskins, 1974; Martin, 1982; Grafton and al., 2012; Michielsen, 2011).

The monopolist's behavior is to maximize the expected present value of revenue from the resource:

$$(3.1) \quad \underset{q_1, q_2 \geq 0}{Max} \{ \pi_1(q_1) + \delta \pi_2(q_2, \alpha q_1) \}$$

$$(3.2) \quad q_1 + q_2 = S$$

The monopolist faces an intertemporal capacity constraint<sup>12</sup>. To justify this, we assume that the resource becomes worthless after the second-period (see Gaudet and al., 1995). It is therefore never optimal to exploit the resource beyond the second period.

Solving the programme above yields the following proposition:

**Proposition 10.** *Monopoly's production of the raw product is optimal if the marginal revenue of period 1 equals the discounted marginal revenue of period 2 net of the effect*

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<sup>11</sup>Stewart (1980) indicates that demand will change over time for two quite different reasons. The first is the standard textbook explanation for shifts in demand: consumer tastes change over time, income rises or falls, substitutes or complements appear, and new households are formed. The second reason is that, if one considers a durable good, its price in any period is a function not only of current and past production, but also of anticipated future production, since that future production will affect future prices (see the paper for more details).

<sup>12</sup>The capacity constraint may simply correspond to the flow of a resource that cannot exceed some level: Thus, if a supplier receives a large order today, he will be constrained on what he can offer in the future (Anton and al., 2014, see also Liski and Montero, 2014). Examples that fit with the concept of capacity constraint include the number of seats on a flight that can be sold before departure, or the number of hotel rooms to be rented for a given night (Martinez-de-Albéniz and Talluri, 2011). The capacity constraint applies also to exhaustible resource markets.

of recycling:

$$(3.3) \quad \underbrace{\frac{\partial \pi_1}{\partial q_1}}_{MR_1} = \delta \underbrace{\frac{\partial \pi_2}{\partial q_2} - \delta \alpha \frac{\partial \pi_2}{\partial r}}_{MR_2}$$

**Proof.** see appendix A □

This result is known as the intertemporal optimization principle which states that discounted marginal revenues should be equalized across periods (see Liski and Montero, 2014). If the resource was exhausted over  $T$  periods, we would have  $MR_1 = MR_2 = MR_3 = \dots = MR_T$ . Proposition 10 indicates that, when there is no recycling, i.e.  $\alpha = 0$ , condition (3.3) echoes the Hotelling rule.

Differentiating the right-hand side of (3.3) with respect to  $\alpha$  yields the following result:

**Proposition 11.** *The effect of recycling on the second-period marginal revenue of the monopolist depends on whether its second-period extraction and recycling are strategic complements or strategic substitutes:*

$$(3.4) \quad \frac{dMR_2^R}{d\alpha} = \underbrace{\delta q_1 \frac{\partial^2 \pi_2}{\partial q_2 \partial r}}_{\text{Strategic effect } (-/+)} + \delta \underbrace{\left[ -\frac{\partial \pi_2}{\partial r} - q_1 \alpha \frac{\partial^2 \pi_2}{\partial r^2} \right]}_{\text{Recycling capacity effect } (+)}$$

(1) *If  $q_2$  and  $r$  are strategic complements, recycling increases monopoly's second-period marginal revenue.*

(2) *Conversely, if  $q_2$  and  $r$  are strategic substitutes, the influence of recycling on monopoly's second-period marginal revenue is ambiguous. It depends on the size of each effect:*

(i) *If the strategic effect is larger than the recycling capacity effect, recycling decreases monopoly's second-period marginal revenue.*

(ii) *In contrast, if the strategic effect is smaller than the recycling capacity effect, recycling increases monopoly's second-period marginal revenue.*

**Proof.** see appendix A □

The intuition behind proposition 11 is as follows. Recycling has two effects on the second-period marginal revenue of the monopoly: a recycling capacity effect which is always positive<sup>13</sup> and a strategic effect<sup>14</sup>. The sign of the strategic effect depends on whether market interactions are characterized by strategic substitutability or strategic complementarity. If the extracted quantity  $q_2$  and the recycled quantity  $r$  are strategic complements, the strategic effect is positive. Consequently, recycling increases the second-period marginal revenues, including that of the monopolist. As will be seen below, the second-period production of the monopoly increases, accordingly, leading to the decrease of its first-period production. However, if the extracted quantity  $q_2$  and the recycled quantity  $r$  are strategic substitutes, the strategic effect becomes negative because the increase of recycling leads the monopolist to reduce its second-period extraction in order to keep the price of the resource high. The effect of recycling depends then on what effect dominates the other. Note that the recycling capacity effect captures the incentives of the monopoly to produce less raw products in the first-period in order to decrease the quantity of recycled product sold on the market in the second-period. If the strategic effect is higher than the recycling capacity effect, the second-period marginal revenue of the monopoly decreases, leading to the drop of its second-period extraction. Therefore, its first-period production increases. Our finding calls into question Gaskins'

<sup>13</sup>  $\frac{\partial \pi_2}{\partial r} < 0$ : higher is the recycled quantity, lower is the second-period profit of the monopolist.

<sup>14</sup> This designation of "strategic effect" is debatable in that we are not completely in the presence of a game. Although the monopolist behaves strategically, such is not the case of the recycler for which the recycling rate is exogenous.

result that Alcoa's first-period production decreases thank to the rise of the price it sets in this period. Conversely, if the recycling capacity effect is higher than the strategic effect, the first-period production of the monopolist decreases thank to the rise of its second-period production. This result is not in line with the earlier literature obtained within the case of the green paradox.

We can summarize the effect of recycling on the first-period production of the raw products through the following proposition:

**Proposition 12.** *The effect of recycling on the first-period production depends on whether raw and recycled products exhibit complementarities or substitutabilities:*

(i) *If raw and recycled products are strategic complements, then recycling decreases first-period production of raw products,  $\partial q_1^*/\partial \alpha < 0$ .*

(ii) *If raw and recycled products are strategic substitutes, then the effect of recycling on the first-period production of raw products is ambiguous.*

**Proof.** see appendix A

□

The intuition underlying point (i) states that if raw and recycled products are strategic complements, the intensification of one strategy (second-period extraction, for instance) triggers the intensification of the other strategy (recycling). Then, the strategic effect becomes positive. Since both effects go in the same direction, recycling increases the second-period marginal revenue of the monopoly which in turn leads it to enhance its second-period production. Since that part of the resource which will not be consumed in the second-period is consumed in the first, the monopolist curbs then its first-period production. One reason for which the monopoly underproduces is that it has the incentive to reduce the ferocity of future competition.



Point (ii) of proposition 12 indicates that if raw and recycled products are strategic substitutes, the strategic effect and the recycling capacity effect go in opposite direction. The intuition is that recycling has two effects on the first-period optimal production of raw products. A remaining question is then whether point (i) can be reversed when raw and recycled products exhibit strategic substitutability?

In order to answer the previous question, let us consider the following specific example:

**Example 1.** Let  $\pi_1(q_1) = p(q_1)q_1$  and  $\pi_2(q_2 + r) = p(q_2 + r)q_2$  where  $p(Q) = 1 - Q$ ,  $S > 0$ ,  $r = \alpha q_1$ ,  $\delta \in (0, 1)$ . In the first-period,  $Q = q_1$  and in the second-period,  $Q = q_2 + r$ . Here, we assume that raw and recycled products are strategic substitutes.

**Proposition 13.** Under example 1 and assuming that the capacity constraint is binding, i.e.  $q_1^* + q_2^* = S$ , recycling increases, always, first-period production of raw products,  $\partial q_1^*/\partial \alpha > 0$ .

**Proof.** See appendix B □

This example shows clearly when there is strategic substitutability, the result obtained in the case of point (i) is completely reversed. In this situation, instead of decreasing the first-period production of raw products, recycling increases it. In fact, if raw and recycled products are strategic substitutes, the strategic effect is always negative and the recycling capacity effect remains positive. But since the first-period production is higher than the second-period production, the strategic effect is larger than the recycling capacity effect, resulting in the decrease of the second-period marginal revenue of the monopolist. The decline of the latter leads the monopoly to reduce the second-period

production of raw products. Consequently, the first-period production of raw products increases, since the resource is depleted over the two periods.

In what follows, we will provide an example where raw and recycled products can exhibit strategic complementarity.

**Example 2.** Let  $\pi_1(q_1) = p(q_1)q_1 - c(q_1)$  and  $\pi_2(q_2 + r) = p(q_2 + r)q_2 - c(q_2)$ , where  $p(q) = 1 - q + \frac{\gamma}{2}q^2$ ,  $c(q) = \frac{1}{2}q^2$  and  $\gamma \leq 1$  and  $q \in [0, 1]$ . Capacity is normalized to unity,  $S = 1$ , and  $\delta = 1$ ,  $q = q_1$  in the first-period and  $q = q_2 + r$  in the second-period.

In this example, the inverse demand function is constant over time and it decreases with the quantity sold,  $p'(q) = -(1 - q) < 0$ . The raw product and the recycled product are perfect substitutes as regards consumers' preferences, as the second-period inverse demand depends on  $q_2 + r$  only. The production cost of the raw product is non decreasing and convex,  $c'(q) = q \geq 0$  and  $c''(q) = 1 > 0$ . Finally, the monopolist does not discount the future.

**Proposition 14.** In example 2, the effect of recycling on the first-period production of raw products depends on the parameter  $\gamma$  :

(1) If  $\gamma > 0$ , recycling decreases the first-period production of raw products, i.e.  $\partial q_1^* / \partial \alpha < 0$

(2) If  $\gamma = 0$ , recycling has no effect on the first-period production of raw products, i.e.  $\partial q_1^* / \partial \alpha = 0$

(3) If  $\gamma < 0$ , recycling increases the first-period production of raw products, i.e.  $\partial q_1^* / \partial \alpha > 0$

**Proof.** see Appendix C

□

If  $\gamma < 0$  ( $\gamma > 0$ ), the second-period marginal revenue for the monopolist decreases (increases) in recycling. Anticipating this decreasing (increasing) effect, the monopolist decreases (increases) its second-period production. Consequently, its first-period production increases (decreases) since the resource constraint is binding. This result is illustrated by figure 3.1 for some values of  $\gamma$ .

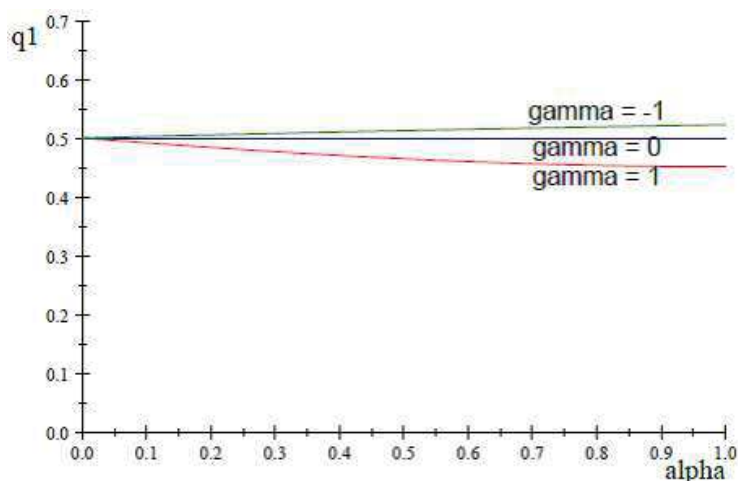


Figure 3.1. The effect of recycling on  $q_1$

This figure shows that point (i) of proposition 12 is reversed, i.e.  $\partial q_1^*/\partial \alpha > 0$ , when the parameter  $\gamma = -1$  (recycling and the raw products exhibit a strong strategic substitutability). In such a context, the strategic effect dominates the recycling capacity effect, leading, respectively, to the decline of both the second-period marginal revenue and the second-period production of the monopoly. A "green paradox" arises in this situation. Conversely, if  $\gamma = 1$  (recycling and extraction are strategic complements). Therefore, the first-period production of the monopoly decreases due to the increase of its second-period marginal revenue and of its second-period production. The traditional result that the presence of a clean backstop leads to the green paradox is reversed in such a situation. Lastly, if  $\gamma = 0$ , the first-period production of the monopoly remains

constant whatever the value taken by  $\alpha \in (0, 1)$ . In this situation, the green paradox does not arise.

### 3.4. Concluding remarks

In this chapter, we have developed a two-period model to analyze the extraction of an exhaustible natural resource in the presence of a competitive recycling sector (or the backstop). Our framework can be used to discuss both the case of Alcoa and the occurrence of a "green paradox". Whereas Gaskins (1974) argues that the market power of Alcoa is not affected by the existence of recycling, our linear model (Example 1) shows the opposite result in the sense that the first-period production of the monopoly increases thanks to recycling. While the earlier literature finds that the long run price of Alcoa is not affected by recycling, our model shows that this result can be reversed when the strategic effect outweighs the recycling capacity effect. This framework suggests also that the traditional result obtained within the context of the green paradox and whereby the presence of a clean backstop can speed up the present extraction of the owner of the natural resource does not always hold. Indeed, when the exhaustible resource and the clean backstop are strategic complements or when the recycling capacity effect is higher than the strategic effect in the case of strategic substitutability, the second-period marginal revenue of the monopoly increases in the presence of the substitute, resulting in the rise of the second-period production of the monopoly. Accordingly, the monopolist decreases its first-period production.

It is worth noting that, in the situation where the strategic effect dominates the recycling capacity effect, the green paradox arises.

The present analysis reverses also some results usually obtained within the context of the capacity constraints by showing that the current production can increase instead

of decreasing and by highlighting that the price of the second-period can take an upward phase under some specified conditions.

We have based our analysis on a two-period model. The challenge for the future is to set up multi-period models or dynamic continuous models in order to show whether our main results still hold.

In the present chapter, we have excluded the possibility that the extractor prevent the entry of the recycling firms. We look at this issue in chapter 4.

### 3.5. Appendix

#### Appendix A: the general model

**Proofs of propositions 10, 11 and 12:** the recycling sector recycles  $\alpha$  at time  $t = 2$ , with  $\alpha$  as a share of  $q_1$ . Then,  $r = \alpha q_1$

The monopolist maximizes the following profit function over the two consecutive periods:

$$(3.5) \quad \underset{q_1, q_2 \geq 0}{Max} \{ \pi_1(q_1) + \delta \pi_2(q_2, r(q_1)) \}$$

$$(3.6) \quad q_1 + q_2 \leq S$$

$$(3.7) \quad r(q_1) = \alpha q_1$$

$$(3.8) \quad \alpha \in [0, 1]$$

The lagrangian for the programme above is given by:

$$L(q_1, q_2, \lambda) = \pi_1(q_1) + \delta \pi_2[q_2, r(q_1)] + \lambda(S - q_1 - q_2)$$

The first order conditions are

$$(3.9) \quad \frac{\partial \pi_1}{\partial q_1} + \delta \alpha \frac{\partial \pi_2}{\partial r} = \lambda$$

$$(3.10) \quad \delta \frac{\partial \pi_2}{\partial q_2} = \lambda$$

$$(3.11) \quad \lambda [S - q_1 - q_2] = 0; \lambda \geq 0$$

By equalizing (3.9) and (3.10), we have:

$$(3.12) \quad \underbrace{\frac{\partial \pi_1}{\partial q_1}}_{MR_1} = \delta \underbrace{\frac{\partial \pi_2}{\partial q_2} - \frac{\partial \pi_2}{\partial r}}_{MR_2^R}$$

where  $-\delta \alpha \frac{\partial \pi_2}{\partial r}$  is an additional term; with  $\alpha = 0$  if there is no recycling. In this situation, the intertemporal optimization principle is given by:

$$(3.13) \quad \underbrace{\frac{\partial \pi_1}{\partial q_1}}_{MR_1} = \delta \underbrace{\frac{\partial \pi_2}{\partial q_2}}_{MR_2^{NR}}$$

From (3.12), we have:

$$(3.14) \quad \frac{dMR_2^R}{d\alpha} = \delta \frac{\partial^2 \pi_2}{\partial q_2 \partial r} \frac{\partial r}{\partial \alpha} - \delta \frac{\partial \pi_2}{\partial r} - \delta \alpha \frac{\partial^2 \pi_2}{\partial r^2} \frac{\partial r}{d\alpha}$$

Where

$$(3.15) \quad \frac{\partial r}{d\alpha} = q_1$$

By rearranging (3.14), we obtain:

$$(3.16) \quad \frac{dMR_2^R}{d\alpha} = \underbrace{\delta q_1 \frac{\partial^2 \pi_2}{\partial q_2 \partial r}}_{\text{Strategic effect } (-/+)} + \delta \underbrace{\left[ -\frac{\partial \pi_2}{\partial r} - q_1 \alpha \frac{\partial^2 \pi_2}{\partial r^2} \right]}_{\text{Recycling capacity effect } (+)}$$

The derivative of (3.12) with respect to  $\alpha$  yields:

$$(3.17) \quad \frac{\partial^2 \pi_1}{\partial q_1^2} * \frac{\partial q_1}{\partial \alpha} = \delta q_1 \frac{\partial^2 \pi_2}{\partial q_2 \partial r} + \delta \left[ -\frac{\partial \pi_2}{\partial r} - q_1 \alpha \frac{\partial^2 \pi_2}{\partial r^2} \right]$$

Which can be transformed in:

$$(3.18) \quad \frac{\partial q_1}{\partial \alpha} = \frac{1}{\frac{\partial^2 \pi_1}{\partial q_1^2}} \left[ \delta q_1 \frac{\partial^2 \pi_2}{\partial q_2 \partial r} + \delta \left( -\frac{\partial \pi_2}{\partial r} - q_1 \alpha \frac{\partial^2 \pi_2}{\partial r^2} \right) \right]$$

$$(3.19) \quad \lim_{\alpha \rightarrow 0} \frac{\partial q_1^*}{\partial \alpha} \propto \lim_{\alpha \rightarrow 0} \frac{1}{\frac{\partial^2 \pi_1}{\partial q_1^2}} \left[ \delta q_1 \frac{\partial^2 \pi_2}{\partial q_2 \partial r} + \delta \left( -\frac{\partial \pi_2}{\partial r} - q_1 \alpha \frac{\partial^2 \pi_2}{\partial r^2} \right) \right]$$

Since  $\pi_1$  is concave in  $q_1$ , we have:  $\frac{\partial^2 \pi_1}{\partial q_1^2} < 0$ . Then,

$$(3.20) \quad \lim_{\alpha \rightarrow 0} \frac{\partial q_1^*}{\partial \alpha} \propto \lim_{\alpha \rightarrow 0} - \left[ \delta q_1 \frac{\partial^2 \pi_2}{\partial q_2 \partial r} + \delta \left( -\frac{\partial \pi_2}{\partial r} - q_1 \alpha \frac{\partial^2 \pi_2}{\partial r^2} \right) \right]$$

$$(3.21) \quad \propto \lim_{\alpha \rightarrow 0} - \left[ \delta q_1 \frac{\partial^2 \pi_2}{\partial q_2 \partial r} + \delta \left( -\frac{\partial \pi_2}{\partial r} \right) \right]$$

$$(3.22) \quad \propto \lim_{\alpha \rightarrow 0} \left[ - \underbrace{\delta q_1 \frac{\partial^2 \pi_2}{\partial q_2 \partial r}}_{\text{Strategic effect } (-/+)} + \underbrace{\delta \frac{\partial \pi_2}{\partial r}}_{\text{Recycling capacity effect } (-)} \right]$$

Note that the recycling capacity effect has a negative effect on the first-period extraction. We can make the following discussion:

(1) If recycling and the second-period extraction are strategic complements, the strategic effect is positive and recycling has a negative effect on the first-period extraction.

(2) Conversely, if recycling and the second-period extraction are strategic substitutes, the strategic effect is negative and the effect of recycling on the first-period extraction is ambiguous and depends on the size of the two effects:

(i) When the recycling capacity effect dominates the strategic effect, recycling decreases the first period extraction

(ii) When the strategic effect dominates the recycling capacity effect, recycling increases the first-period extraction.



**Appendix B: raw and recycled products are considered to be strategic substitutes**

**Proof of proposition 13:** under example 1, the programme of the monopolist is given by:

$$(3.23) \quad \underset{q_1, q_2 \geq 0}{Max} \{ (a - bq_1)q_1 + \delta(a - b(q_2 + \alpha q_1))q_2 \}$$

$$(3.24) \quad q_1 + q_2 \leq S$$

$$(3.25) \quad r(q_1) = \alpha q_1$$

$$(3.26) \quad \alpha \in [0, 1]$$

The lagrangian for the programme above is given by:

$$(3.27) \quad L(q_1, q_2, \lambda) = (a - bq_1)q_1 + \delta(a - b(q_2 + \alpha q_1))q_2 + \lambda(S - q_1 - q_2)$$

The first order conditions are

$$(3.28) \quad a - 2bq_1 + \delta(-b\alpha q_2) = \lambda$$

$$(3.29) \quad \delta(a - b(q_2 + \alpha q_1) - bq_2) = \lambda$$

$$(3.30) \quad \lambda(S - q_1 - q_2) = 0, \lambda > 0$$

According to the intertemporal optimization principle, discounted marginal revenues should be equalized accross periods, that is,

$$(3.31) \quad \underbrace{a - 2bq_1}_{MR_1} = \underbrace{\delta(a - b(q_2 + \alpha q_1) - bq_2 + b\alpha q_2)}_{MR_2}$$

(i) **Effect of recycling on the second-period marginal revenue of the monopolist:** it is given by

$$(3.32) \quad \frac{dMR_2}{d\alpha} = \underbrace{-\delta bq_1}_{\text{Strategic effect (-)}} + \underbrace{b\delta q_2}_{\text{Recycling capacity effect (+)}}$$

Because the profit function is identical in period 1 and 2; recycling (strategic substitutability) and discounting:

$$(3.33) \quad q_1^* > q_2^*$$

Hence,

$$(3.34) \quad \frac{dMR_2}{d\alpha} = \delta b (q_2 - q_1) < 0$$

(ii) **Calculation of the optimal quantities when the resource constraint is binding**

In this situation,  $\lambda > 0$  and  $S - q_1 - q_2 = 0$ . Therefore,  $q_2 = S - q_1$ . We have, then, the following system of equations

$$(3.35) \quad a - 2bq_1 + \delta(-b\alpha q_2) = \lambda$$

$$(3.36) \quad \delta(a - b(q_2 + \alpha q_1) - bq_2) = \lambda$$

Substitute  $q_2=S - q_1$  in (3.35) and (3.36) yields:

$$(3.37) \quad a - 2bq_1 + \delta(-b\alpha(S - q_1)) = \lambda$$

$$(3.38) \quad \delta(a - b(S - q_1 + \alpha q_1) - b(S - q_1)) = \lambda$$

By equalizing the two equations above, we have:

$$q_1^* = \frac{a(1 - \delta) + bS\delta(2 - \alpha)}{b(2 - \alpha\delta + \delta(2 - \alpha))}$$

$$\frac{dq_1^*}{d\alpha} = -\frac{1}{2b}\delta(\delta - 1) \frac{a - Sb}{(\delta - \alpha\delta + 1)^2} > 0$$

In addition, we have:

$$(3.39) \quad q_2^* = \frac{-a(1 - \delta) + bS(2 - \alpha\delta)}{b(2 - \alpha\delta + \delta(2 - \alpha))}$$

$$(3.40) \quad \frac{dq_2^*}{d\alpha} = \frac{1}{2b}\delta(\delta - 1) \frac{a - Sb}{(\delta - \alpha\delta + 1)^2} < 0$$

### Appendix C: raw and recycled products may exhibit strategic complementarity

**Proof of proposition 14:** Under example 2, the programme of the monopoly is given by:

$$(3.41) \quad \max_{q_1, q_2 \geq 0} \left\{ (1 - q_1 + \frac{\gamma}{2}q_1^2)q_1 - \frac{1}{2}q_1^2 + \left( 1 - (q_2 + \alpha q_1) + \frac{\gamma}{2}(q_2 + \alpha q_1)^2 \right) q_2 - \frac{1}{2}q_2^2 \right\}$$

s.t.

$$(3.42) \quad q_1 + q_2 \leq 1$$

The lagrangian for this programme is given by:

$$(3.43) \quad L(q_1, q_2, \lambda) = (1 - q_1 + \frac{\gamma}{2}q_1^2)q_1 - \frac{1}{2}q_1^2 + \left(1 - q_2 - \alpha q_1 + \frac{\gamma}{2}(q_2 + \alpha q_1)^2\right)q_2 - \frac{1}{2}q_2^2 + \lambda(1 - q_1 - q_2)$$

The necessary conditions are given by:

$$(3.44) \quad \frac{\partial L}{\partial q_1} = \gamma\alpha^2q_1q_2 + \gamma\alpha q_2^2 - \alpha q_2 + \frac{3}{2}\gamma q_1^2 - 3q_1 - 1 = \lambda$$

$$(3.45) \quad \frac{\partial L}{\partial q_2} = \frac{1}{2}\gamma\alpha^2q_1^2 + 2\gamma\alpha q_1q_2 - \alpha q_1 + \frac{3}{2}\gamma q_2^2 - 3q_2 - 1 = \lambda$$

$$(3.46) \quad \lambda(1 - q_1 - q_2) = 0, \quad \lambda \geq 0$$

(i) **Calculus of the second-period marginal revenue of the monopolist:**

$$(3.47) \quad MR_2 = \frac{1}{2}\gamma\alpha^2q_1^2 + 2\gamma\alpha q_1q_2 - \alpha q_1 + \frac{3}{2}\gamma q_2^2 - 3q_2 + 1$$

$$(3.48) \quad \frac{dMR_2}{d\alpha} = q_1[\gamma(2q_2 + \alpha q_1) - 1]$$

• Equation (3.48) shows clearly that if  $\gamma < 0$ ,

$$(3.49) \quad \frac{dMR_2}{d\alpha} < 0$$

We can rewrite (3.48) as follows:

$$(3.50) \quad \frac{dMR_2}{d\alpha} = q_1 \left( \underbrace{2\gamma q_2}_+ + \underbrace{\alpha\gamma q_1 - 1}_- \right)$$

- When  $\gamma > 0$ ,  $\frac{dMR_2}{d\alpha} > 0$  if and only if

$$(3.51) \quad q_1^* < \frac{2\gamma - 1}{\gamma(2 - \alpha)}$$

Since  $q_1^* > 0$ , then we have:

$$(3.52) \quad \gamma > \frac{1}{2}$$

Let us verify if  $q_1^* < \frac{2\gamma - 1}{\gamma(2 - \alpha)}$ . If so,

$$(3.53) \quad \frac{(3 - \alpha)(\gamma(\alpha - 1) + 2) - \sqrt{\alpha(4\alpha + 24\gamma - 6\gamma^2 - 2\alpha^2\gamma^2 + \alpha^3\gamma^2 + 2\alpha\gamma + \alpha\gamma^2 - 2\alpha^2\gamma - 24) + 9(\gamma - 2)^2}}{3\alpha\gamma(2 - \alpha)} < \frac{2\gamma - 1}{\gamma(2 - \alpha)}$$

The previous inequation can be written as follows:

$$-\sqrt{\alpha(4\alpha + 24\gamma - 6\gamma^2 - 2\alpha^2\gamma^2 + \alpha^3\gamma^2 + 2\alpha\gamma + \alpha\gamma^2 - 2\alpha^2\gamma - 24) + 9(\gamma - 2)^2} < \frac{(3 - \alpha)(\gamma(\alpha - 1) + 2) - 3\alpha(2\gamma - 1)}{\gamma(2 - \alpha)}$$

The previous inequation turns into:

$$(3.55) \quad \frac{\overbrace{\alpha - 3\gamma - 2\alpha\gamma - \alpha^2\gamma + 6}^A}{\sqrt{\alpha(4\alpha + 24\gamma - 6\gamma^2 - 2\alpha^2\gamma^2 + \alpha^3\gamma^2 + 2\alpha\gamma + \alpha\gamma^2 - 2\alpha^2\gamma - 24) + 9(\gamma - 2)^2}} < 0$$

It is straightforward to notice that it is a second degree inequation in  $\gamma$ . Solving it yields two roots:

$$(3.56) \quad \gamma_1 = \frac{3\alpha + 9 - \sqrt{24\alpha - 12\alpha^2 + 2\alpha^3 + 9}}{3\alpha + 2\alpha^2 + 6}$$

$$(3.57) \quad \gamma_2 = \frac{3\alpha + 9 + \sqrt{24\alpha - 12\alpha^2 + 2\alpha^3 + 9}}{3\alpha + 2\alpha^2 + 6}$$

Since the dicriminant is positive, the expression is negative between the two roots. It is also easy to show that  $\gamma_2 > 1$ . Then, this root is to preclude. Now, let us show that  $\gamma_1 > \frac{1}{2}$ . That is the case if

$$(3.58) \quad (2\alpha - 9)(\alpha - 2)(3\alpha + 2\alpha^2 + 6) > 0$$

The expression above holds if  $2\alpha - 9 < 0$  (true because  $\alpha < 1$ ). We conclude that  $\gamma \in (\frac{1}{2}, 1)$ . Then  $A < 0$ . Hence  $q_1^* < \frac{2\gamma-1}{\gamma(2-\alpha)}$ . Therefore

$$(3.59) \quad \frac{dMR_2}{d\alpha} > 0$$

**(ii) Calculus of the first-period production of the monopolist:** Since we assume the resource is exhausted over the two periods,  $1 - q_1 - q_2 = 0$ , then  $\lambda > 0$  and  $q_2 = 1 - q_1$ .

By equalizing equations (3.44) and (3.45) and by considering that  $q_2 = 1 - q_1$ , we have:  $\alpha\gamma - 3q_1 - \alpha + \alpha q_1 + \frac{3}{2}\gamma q_1^2 + \alpha\gamma q_1^2 + \alpha^2\gamma q_1 -$

$$(3.60) \quad \alpha^2\gamma q_1^2 - 2\alpha\gamma q_1 - 1 = \frac{3}{2}\gamma + 3q_1 - \alpha q_1 - 3\gamma q_1 + \frac{3}{2}\gamma q_1^2 - 2\alpha\gamma q_1^2 + \frac{1}{2}\alpha^2\gamma q_1^2 + 2\alpha\gamma q_1 - 4$$

Whose the rearrangement gives:

$$(3.61) \quad \alpha\gamma - \frac{3}{2}\gamma - 6q_1 - \alpha + 2\alpha q_1 + 3\gamma q_1 + 3\alpha\gamma q_1^2 + \alpha^2\gamma q_1 - \frac{3}{2}\alpha^2\gamma q_1^2 - 4\alpha\gamma q_1 + 3 = 0$$

Solving (5.9) yields:

$$\text{case 1: } -3\alpha\gamma + \frac{3}{2}\alpha^2\gamma \neq 0$$

Let us see whether  $-3\alpha\gamma + \frac{3}{2}\alpha^2\gamma \neq 0$  or not. This holds if  $\alpha \neq 0$  (this is always true if there is recycling) or  $\alpha \neq 2$  (this is always verified because  $\alpha \leq 1$ ). Then,  $-3\alpha\gamma + \frac{3}{2}\alpha^2\gamma \neq 0$  and the first-period quantity is given by (recall that the other root is precluded because it is not consistent with our assumptions: either it is negative, either it is not defined on  $\alpha \in (0, 1)$ ):

$$(3.62) \quad q_1^* = - \frac{\left( \frac{(\alpha - 3)[\gamma(\alpha - 1) + 2]}{\alpha(4\alpha + 24\gamma - 6\gamma^2 - 2\alpha^2\gamma^2 + \alpha^3\gamma^2 + 2\alpha\gamma + \alpha\gamma^2 - 2\alpha^2\gamma - 24) + 9(\gamma - 2)^2} \right)}{3\alpha\gamma(2 - \alpha)}$$

The first-period quantity can be plotted for some values of  $\gamma$ :

► If  $\gamma = 1$ ,

$$(3.63) \quad q_1^* = \frac{1}{3\alpha(2 - \alpha)} \left( 2\alpha - \alpha^2 - \sqrt{\alpha^4 - 4\alpha^3 + 7\alpha^2 - 6\alpha + 9} + 3 \right)$$

► If  $\gamma = -1$ ,

$$(3.64) \quad q_1^* = \frac{1}{3\alpha(2 - \alpha)} \left( 6\alpha - \alpha^2 + \sqrt{\alpha^4 + 3\alpha^2 - 54\alpha + 81} - 9 \right)$$

**case 2:**  $-3\alpha\gamma + \frac{3}{2}\alpha^2\gamma = 0 \wedge -2\alpha - 3\gamma + 4\alpha\gamma - \alpha^2\gamma + 6 \neq 0$ , then:

$$(3.65) \quad q_1^* = \frac{-\alpha(\gamma - 1) + \frac{3}{2}(\gamma - 2)}{(\alpha - 3)(\gamma(\alpha - 1) + 2)}$$

► If  $\gamma = 0$

$$(3.66) \quad q_1^* = \frac{1}{2}$$



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## CHAPTER 4

**Strategic Extraction in front of Recycling****Abstract**

We examine the extractor's best strategies for an exhaustible resource that is recycled by an independent competitive company with fixed costs upon entry. Our findings provide insight on the possibility of socially inefficient extraction of the virgin resource. When recycling is relevant, the first-best solution requires to accommodate or promote recycling by increasing prior extraction since the recycled material generates additional resources. The monopolist-extractor, however, sees recycling as a threat and hence, it strategically chooses prior extraction to influence the future price of the resource. Specifically, the monopolist will either increase or decrease prior extraction in equilibrium, depending on whether it wishes to deter or to accommodate recycling. We also examine the effects of resource scarcity and fixed costs magnitudes on the extraction of the virgin resource.

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**Keywords:** Entry, Exhaustible Resource, Monopolist, Recycling.

**4.1. Introduction**

Since the early work of Smith (1972), recycling waste or scrap into production has been viewed as an alternative to undesirable littering. It is also commonly recognized that recycling helps save natural resources through conservation. This basic idea however needs to be addressed regarding those exhaustible resources, such as phosphorus or aluminum, that can be partially recovered after use. Surprisingly enough, the economics

literature on exhaustible resources has not much considered the possibility of recycling. Although developments in recycling techniques have raised the effective stocks of resources (see Dasgupta, 1993), it is not clear why recycling should curb the extraction of resources that tend to run out.

Our goal is to investigate the effects of recycling on the extraction of an exhaustible resource. We examine the strategic interaction between a resource extractor and an independent competitive recycler in a two-period model where the recycler incurs fixed costs upon entry. The analysis compares the virgin resource extraction prior to recycling when the extractor is a monopolist with the extraction that would be desirable by a social planner who takes into account both the value created by the recycler and that derived from extraction. The model assumes that recycling yields a perfect substitute for the virgin resource. On one side, recycling generates additional resources, which is socially desirable, especially when the resource is scarce. But on the other side, a monopolist-extractor views the recycler's entrant as a threat to its market power. The monopolist anticipates how its initial choice of extraction affects not only the present and future demands for the resource, but also the intensity of future competition with the recycler. Even though recycling is socially desirable, the monopolist may find it more profitable to prevent the recycler's entry under certain circumstances. Or, if the recycler is entering in any case, the arbitrage rule of Hotelling (1931) that the monopolist's marginal revenue from extraction rises at the rate of interest must be amended to take recycling into consideration. In a nutshell, resource extraction has a commitment value that signals to potential recyclers whether the monopolist-extractor will prevent or restrict competition against them. We characterize the equilibrium choice of prior extraction, depending on whether it is made by the social planner or the monopolist. We carry out some comparative statics to examine how various changes in the underlying parameters of

recycling costs and resource scarcity affect prior extraction and hence the recycling possibilities.

As a result, the first-best solution requires the extraction sector to let the recycling company enter the market, provided that the resource is scarce enough and fixed costs of recycling are low enough. In the first-best outcome, the invitation to recycle can take two different forms depending on the fixed cost magnitudes. If the market is attractive enough to the recycling company because of significantly low fixed costs, the extraction sector must *accommodate* recycling by increasing prior extraction above the level prevailing with no possibility of recycling. Then, the resource price is rising more rapidly than the interest rate at the first-best, because prior extraction generates additional resources via recycling. Hence, accommodating recycling disrupts the standard rule of Hotelling that price is rising at the interest rate. In contrast, for higher fixed costs, the extraction sector must reduce prior extraction to encourage the recycling company to enter, thereby *promoting* recycling.

In the monopolist's outcome, however, the opposite may happen because recycling is perceived as a threat to future profits: the monopolist strategically chooses prior extraction to discourage recycling. For this, the monopolist may implement two slightly different strategies: either the monopolist will *ignore* recycling, thereby behaving as if recycling were irrelevant, if the resulting downward pressure on the future price of the resource is enough to make the market unattractive to the recycling company; or the monopolist will *deter* recycling by raising prior extraction above the level prevailing with no recycling to push the future price down far enough that the recycling company stays out. Recycling deterrence is the monopolist's best strategy when the fixed costs of recycling are not too high.

However, if both the fixed costs are so low and the resource is so scarce that recycling cannot be avoided, we find that the monopolist also accommodates recycling in equilibrium. In that case, the arbitrage rule of Hotelling is disrupted again: the monopolist extracts strategically little prior to recycling— actually less than what would be extracted with no recycling —to soften future competition between recycling and extraction.

The chapter is organized as follows. Section 4.2 presents a detailed review of the related literature. Section 4.3 introduces the two-period model. Section 4.4 presents the first-best solution. In Section 4.5, we analyze the case of a monopolist in the resource extraction sector faced with an independent competitive company in the recycling industry. Concluding remarks appear in Section 4.6.

## 4.2. Related literature

The history of exhaustible resources shows evidence that the extraction sector goes through various regimes of competition and the recycling market is often ill-organized. Martin (1982) recognizes that “many of the industries currently practicing recycling are highly concentrated”.

One interesting example is phosphate extraction together with phosphorus recycling. The majority of global phosphate rock reserves are located in Morocco, providing this country with a monopoly position in supplying the virgin resource (see Cordell et al., 2009). Thus, one may expect governmental regulation in Morocco to play a leading role in choosing the quantity of virgin phosphate to be extracted. In turn, this regulation may be more or less benevolent, depending on various factors such as the pressure put on the government by shareholders of the extraction company, or the share of the consumer surplus that escapes the government’s jurisdiction. At the same time, the sector of phosphorus recycling has no institutional or organizational home (Cordell et



al., 2006; Livingston et al., 2005). Phosphorus recycling throughout the world is mainly based on the reuse of nutrient flows stemming from food production and consumption<sup>1</sup>. While the sanitation sector in cities, e.g. waste water treatment or sewage sludge plants, plays a key role in phosphorus recycling<sup>2</sup>, this service is scarcely high on the agenda of extraction stakeholders. In addition, the process of recovering phosphorus from sewage or waste water often requires a specific infrastructure and high levels of technical skills. According to Weikard and Seyhan (2009), phosphorus recycling is mainly undertaken by developed countries, except for Pakistan, not only because they have advanced wastewater treatment technologies, but also because, unlike developing countries, they have phosphorus-saturated soils<sup>3</sup>.

Another example of a recyclable exhaustible resource is aluminum. This is now well documented because aluminum has been recovered since the early 1900s<sup>4</sup>. The monopolistic nature of virgin aluminum production in 1945 was acknowledged by the famous Alcoa case (Swan, 1980<sup>5</sup>). In contrast, the recycling sector of the industry is generally considered as competitive throughout the literature. In the view of Friedman (1967), the competitive recycling company would tend to push the aluminum price down to the

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<sup>1</sup>There are various methods available to recover phosphorus, such as ploughing crop residues back into the soil, composting food waste from households, using human and animal excreta, etc.

<sup>2</sup>“Around 41% of phosphorus from sewage sludge across the European Union is currently recovered and reused in agriculture”—the European Commission’s expert seminar on the sustainability of phosphorus resources (2011, [http://ec.europa.eu/environment/natres/pdf/conclusions\\_17\\_02\\_2011.pdf](http://ec.europa.eu/environment/natres/pdf/conclusions_17_02_2011.pdf).) even now, according to Ensink et al. (2004), more than 25% of urban vegetables grown in Pakistan are being fertilized with municipal wastewater.

<sup>3</sup>These authors show that developing countries benefit in the short and medium run from phosphorus recycling in developed countries, but face stronger competition for the resource in the long-term.

<sup>4</sup>In 1989, about 28% of the total aluminum supply in the United States came from recovered aluminum (see <http://www.epa.gov/osw/nonhaz/municipal/pubs/sw90077a.pdf>).

<sup>5</sup>In 1945, Alcoa was judged to enjoy a strong monopoly position which was supported rather than threatened by competition from secondary aluminum, produced by recycling scrap aluminum. Swan (1980) provides empirical evidence that the price charged by Alcoa is only slightly below the pure monopoly price but is well above the purely competitive price. The question of whether Alcoa had maintained its monopoly position by strategically controlling the supply of scrap aluminum ultimately available to secondary producers has been debated at length in the economic literature. Grant (1999) provides a nice survey of this debate.

marginal cost of virgin aluminum production. Martin (1982) disputes this statement in a model where Alcoa is treated as a monopolist faced with an independent recycling company. Assuming that a fixed proportion of scrap is discarded by consumers, that author shows that the long run price sold by the monopolist is strictly greater than the marginal cost of virgin aluminum. Suslow (1986) argues that Alcoa's market power was barely eroded by the very competitive nature of recycling, because virgin and recovered aluminum were not perfect substitutes. This view conflicts with Swan (1980)'s intuition that the monopolist in the aluminum extraction sector had a strong strategic control over the recycling industry. Building on the assumption that the two sectors of extraction and recycling were independent in the Alcoa case, Grant (1999) provides empirical evidence that, first, recycling mattered to Alcoa, second, the producer of the virgin resource enjoyed a significant degree of market power, and third, aluminum recycling was not efficient although the sector was competitive. Since then, the aluminum industry has gone through different regimes of imperfect competition, both in the extraction and the recycling sectors.

The early theoretical literature related to this chapter has examined how market power in the extraction sector affects the Hotelling rule. Hotelling (1931) shows that the monopolist has a tendency to be more resource-conservative than "competition... or maximizing of social value would require". Stiglitz (1976) adds that the parsimony of the monopolist depends on the elasticity of demand and extraction costs. Except for the case where the elasticity of demand is constant and extraction costs are zero, the result that the monopolist extracts the resource at a lower rate than that of the competitive firm seems rather robust (see also Tullock, 1979, for the case of inelastic demand). Lewis (1975) however discovers conditions on the price elasticity of demand for which the monopolist depletes the resource faster than required by social efficiency.

Furthermore, a growing number of Cournot competitors on the market for an exhaustible resource tends to increase early extraction (see Lewis and Schmalensee, 1980). Hoel (1978) analyzes a situation in which the monopolist in the extraction sector faces perfect competition with a perfect substitute for the exhaustible resource, and shows that the monopolist reduces initial extraction compared to the case where the monopolist controls both resource extraction and substitute production. In the present analysis, substitute production results from prior extraction, hence the extraction sector determines the amount of input available for substitute production.

The issue of recycling an exhaustible resource has developed more recently in the economic literature with the aforementioned debate on the Alcoa case. Besides that, Hollander and Lasserre (1988) investigate the case of a monopolist in the extraction sector which recycles the scrap from its own production. The monopolist has monopsony power in the scrap market and faces a fringe of price-taking recyclers. Those authors show that the extraction sector finds it profitable to preempt market entry by competitive recyclers when the cost of recycling is sufficiently high. In contrast, in the present chapter we analyze the competition between the virgin resource and the recycled product that occurs after prior extraction, assuming that the extraction sector does not recycle its own output. Gaudet and Van Long (2003) examine how market power in the recycling industry affects the primary production of a non-exhaustible resource. They show that the possibility of recycling may increase the market power of the extraction sector. Clearly, this cannot occur in the present model since competition between the exhaustible resource and its recycled output mitigates the extraction sector's market power. Lastly, Fisher and Laxminarayan (2004) demonstrate that a monopolist may extract the exhaustible resource faster than a competitive company when the resource

is sold at different prices on two separate markets with different iso-elastic demands and no arbitrage possibility between the markets.

### 4.3. The two-period model

In a market for an exhaustible natural resource, an extraction sector is facing one prospective recycling company, which must decide whether to enter the market. The extraction sector, indexed by  $i = 1$ , holds the stock of the exhaustible natural resource, equal to  $s$ . This sector can extract the resource and transport it to market at no cost. Exploration does not occur and  $s$  is the single known stock of the resource in the world of this model. The exhaustible resource market is characterized by an inverse demand function  $P(q)$ , hence the consumers' gross surplus is  $S(q) = \int_0^q P(x)dx$ . We will assume that  $P(q)$  is twice continuously differentiable with  $P'(q) < 0$ .

The independent recycling company, indexed by  $i = 2$ , has the technology and skill to recover part of the resource from used quantities<sup>6</sup>. The buyers of the virgin resource dispose of the used resource within the recycling industry, e.g., because it cannot be used again without being recycled. The recycled resource is viewed by consumers as a perfect substitute for the extracted resource. Recycling the amount  $q$  of the extracted resource yields an output  $r$  that cannot exceed  $q$  due to the depreciation and shrinkage which are present in every recovery process<sup>7</sup>. Should the recycling company decide to enter, it must incur a set-up cost of  $F$  and the recycling technology is given by the cost function  $c(r) = cr$ , where the constant marginal costs  $c$  reflect the value of the used virgin resource together with the prices of all the factors needed to produce the recovered substitute of the resource.

<sup>6</sup>Regarding phosphorus, for instance, sector 2 may be viewed as the group of developed countries with phosphorus-saturated soils and advanced wastewater treatment technologies (see Weikard and Seyhan, 2009).

<sup>7</sup>See Martin (1982) for aluminum scrap recovery and Weikard and Seyhan (2009) for phosphorus recovery from sewage sludge.

We model the extraction process and the entry decision of the recycling company as a two-period game. This implies that the resource becomes worthless after two periods. The extraction sector divides the resource stock between both periods. Supply in the first period determines what is left to be sold in the second period. In the first period, the extraction sector chooses quantity  $q$  and the market clears at price  $P(q)$ . In the second period, the recycling company decides whether to enter the market. If entry occurs, the recycling company produces quantity  $r$  and, simultaneously, the remaining stock of the resource,  $s - q$ , is sold by the extraction sector; the market then clears at price  $P(s - q + r)$ . The recycling company is assumed to be perfectly competitive.

The objective of the extraction sector is to maximize the objective function

$$(4.1) \quad W^1 = \eta(S_1 - \pi_1^1) + \pi_1^1 + \delta [\eta(S_2 - \pi_2^1) + \pi_2^1]$$

where  $\delta$  is the discount factor,  $S_1 = S(q)$ ,  $\pi_1^1 = P(q)q$ ,  $S_2 = S(s - q + r)$ ,  $\pi_2^1 = P(s - q + r)(s - q)$  and  $\eta \in \{0, 1\}$ .

The objective function for the recycling company is given by

$$(4.2) \quad W^2 = S_2 - cr - F,$$

In the economies we have in mind, the recycling industry is similar to a fringe of small price-taking firms and the extraction sector either exercises monopoly power within its own business ( $\eta = 0$ ) or behaves as a social planner ( $\eta = 1$ ) who internalizes the value created by the recycling company in addition to taking into account the consumer surplus derived from virgin resource extraction. The social planner's outcome obtained with  $\eta = 1$  will set the benchmark. The case  $\eta = 0$  is motivated by real-world features of the phosphorus and aluminum industries. The market for phosphorus is mainly characterized by high concentrations of phosphate reserves in a few countries, such as Morocco and

China (see Cordell et al., 2009, or Weikard and Seyhan, 2009). The case  $\eta = 0$  is also closely related to Swan (1980)'s study of the market for aluminum, where the monopolist "Alcoa" is confronted by an independent competitive recycling company (see also Martin, 1982).

In the absence of recycling, we denote by  $q_0^e$  the socially efficient first-period resource extraction, in the sense that

$$(4.3) \quad P(q_0^e) = \delta P(s - q_0^e)$$

To ensure that, under perfect competition, the extraction sector is active in the absence of recycling, we will make the following assumption

$$(4.4) \quad P(q_0^e) > 0$$

Since the first-period resource extraction determines what is left to be sold in the second period, the size of the stock constrains the extraction sector, which thus takes no strategic decision in the second period. The prior extraction decision is irrevocable: it has a commitment value, which influences the recycling company's decision. The recycling company observes the first-period extraction  $q$ , and decides whether to enter the market or to stay out. We normalize the welfare secured by the recycling company if it stays out to be zero. Thus, the recycling company becomes active if and only if it satisfies a participation constraint requiring that the social welfare  $W^2$  exceeds zero. A (pure) strategy for the extraction sector is a choice  $q$ , and a strategy for the recycling company is a mapping  $R : [0, +\infty) \rightarrow [0, +\infty)$ . It follows that the equilibrium of the two-period entry game reduces to a pair  $(q^*, R(\cdot))$  of Nash equilibrium with sequential move defined as follows:

1.  $W^1(q^*, R(q^*)) \geq W^1(q, R(q))$ , for all  $q \in [0, s]$ ;

2.  $W^2(q^*, R(q^*)) \geq W^2(q^*, r)$ , for all  $r \in [0, s]$ ,

subject to  $W^2(q^*, R(q^*)) \geq 0$ .

This means that the extraction sector, by its initial commitment, can decide whether the recycling company enters the market or not. The participation constraint ensures that the recycling company finds it worthwhile to enter. In the case of entry, the extraction sector chooses a point on the recycling company's reaction function to maximize its own welfare.

To solve this game, the first step is to derive the subgame reaction function of the recycling company to the level  $q$  of prior extraction. The recycling company maximizes

$$(4.5) \quad W^2(q, r) = S(s - q + r) - cr - F.$$

We denote the recycling company's reaction function by  $R(q)$ . We neglect scales economies for a moment and concentrate on the levels of  $q$  that allow the recycling company to enter the market. In that case,  $R(q)$  coincides with the output  $\tilde{r}(q)$  at which the market price equals the marginal cost of recycling,

$$(4.6) \quad P(s - q + \tilde{r}(q)) = c.$$

To get the existence (and unicity) of  $\tilde{r}(q)$ , it is sufficient that  $P(q)$  be log-concave<sup>8</sup>. Hence,  $\tilde{r}(q)$  represents the optimal level of recycling whenever possible. One key feature of recycling is that full recycling is impossible. We will assume<sup>9</sup>

$$(4.7) \quad W_r^2(q, q) < 0,$$

<sup>8</sup> $P(q)$  is log-concave if  $P''(\cdot)P(\cdot) - P'(\cdot)^2 < 0$ . This condition is satisfied when  $P$  is concave, linear or  $P(q) = Aq^{\gamma-1}$  with  $0 < \gamma < 1$  so that  $1/(1-\gamma)$  is the elasticity of demand. Most of the commonly used demand functions are, in fact, log-concave. The limiting case is  $P(q) = Ae^{-q}$ , which is strictly convex and log-linear (hence log-concave). When  $P(q)$  is log-concave, the recycling company's problem is concave.

<sup>9</sup>Throughout the article, a subscript will denote a derivative with respect to the relevant variable.

which amounts to  $P(s) < c$ , so that the output of recycling  $\tilde{r}(q)$  always falls short of the output  $q$  previously extracted. At  $q = 0$ , (4.7) implies  $W_r^2(0, 0) < 0$ . As  $W_r^2(0, \tilde{r}(0)) = 0 > W_r^2(0, 0)$  and  $W_{rr}^2(q, r) = P'(s - q + r) < 0$ , we also have  $\tilde{r}(0) < 0$ . Furthermore, differentiating  $W_r^2(q, \tilde{r}(q)) = 0$ , we get

$$(4.8) \quad \tilde{r}'(q) = -\frac{W_{rq}^2(\cdot)}{W_{rr}^2(\cdot)} = 1.$$

As  $\tilde{r}(q)$  is upward sloping, there exists  $\underline{q} > 0$  such that  $\tilde{r}(\underline{q}) = 0$ , hence  $\underline{q}$  is the minimum level of prior extraction that accommodates recycling. For recycling to be effective, we need that  $\underline{q} < s$ . For this, we assume further

$$(4.9) \quad W_r^2(s, 0) > 0,$$

which amounts to  $P(0) > c$ , so that  $W_r^2(s, 0) > W_r^2(s, \tilde{r}(s))$  implies  $\tilde{r}(s) < 0$  since  $W_r^2(s, q)$  is strictly decreasing, and thus  $\tilde{r}(s) < \tilde{r}(\underline{q})$ . We see that extracting more of the resource in the first period induces the recycling company to produce more in the next period, provided that prior extraction allows the recycling activity. Hence, prior extraction creates the recycling activity, which yields a perfect substitute to the virgin resource produced by future extraction. Thus, increasing prior extraction generates additional resources via recycling and, at the same time, expands the future market share for the recycled substitute, which in turn reduces the future market share for the virgin resource.

We now introduce scale economies. Let  $\tilde{q}$  be the level of  $q$  (higher than  $\underline{q}$ ) that makes the recycling company indifferent between staying out and entering, so that  $R(\tilde{q}) = \tilde{r}(\tilde{q})$  and  $W^2(\tilde{q}) = 0$ , where  $\mathbf{W}^2(q) = W^2(q, R(q))$  is the reduced-form function. The recycling company's reaction function is discontinuous at the level  $\tilde{q}$ , where there is a jump of the same sign as  $\left. \frac{d\mathbf{W}^2(q)}{dq} \right|_{q=\tilde{q}}$ . The recycling reaction function is made up of two possible



segments within  $[q, s]$ . One segment corresponds to  $R(q) = 0$ , meaning that the recycling company is better off securing zero welfare. The other segment includes all the levels  $q$  that allows the recycling company to enter and produce  $R(q) = \tilde{r}(q)$ . The position of the discontinuity depends on the underlying parameters of demand and recycling cost. From the envelope theorem, we can write  $\frac{d\mathbf{W}^2(q)}{dq} = W_q^2(q, R(q))$ , and so

$$(4.10) \quad \frac{d\mathbf{W}^2(q)}{dq} \Big|_{q \geq \underline{q}} = -P(s - q + \tilde{r}(q)).$$

As the sign of the derivative of  $W^2(q)$  is negative for all  $q \geq \underline{q}$ , the recycling reaction function is downwards jumping at  $\tilde{q}$ . Increasing prior extraction above  $\tilde{q}$  prevents entry because it reduces the second-period consumer surplus derived from virgin resource extraction by  $P(\cdot)$  for all the units of extracted resource. Hence,  $\tilde{q}$  is the maximum level of prior extraction below which the recycling company enters the market, choosing the output  $\tilde{r}(q)$ . Formally, the recycling reaction function is

$$(4.11) \quad R(q) = \begin{cases} \tilde{r}(q) & \text{when } \underline{q} \leq q \leq \tilde{q}, \\ 0 & \text{otherwise.} \end{cases}$$

Anticipating(4.11), the extraction sector chooses  $q$  to maximize the reduced-form function  $\mathbf{W}^1(q) = W^1(q, R(q))$ . As  $R(q)$  is discontinuous at  $\tilde{q}$ ,  $\mathbf{W}^1(q)$  is also discontinuous at  $\tilde{q}$ . Thus,  $\mathbf{W}^1(q)$  is not concave in  $q$ , and it may achieve multiple local maxima, of which one accommodates the recycling company and the other does not. Let  $q^a$  denote the local maximum that accommodates the recycling company. It must satisfy the first-order condition

$$(4.12) \quad W_q^1(q, \tilde{r}(q)) + W_r^1(q, \tilde{r}(q))\tilde{r}'(q) = 0,$$

where  $\tilde{r}'(q) = 1$  from (4.8). The total derivative of the extraction sector's welfare in the left-hand side of (4.12) gives the incentive to extract the resource prior to recycling. It can be decomposed into two effects. The first effect is  $W_q^1$ . This is a "balance effect" between the first and the second period: any welfare improvement produced in the first period by the extraction of the virgin resource is offset by a welfare deterioration in the second period. The balance effect would exist even if prior extraction of the resource were not recovered, and therefore recycling could not affect future extraction. The second effect, captured by  $W_r^1$ , is a "recycling effect" that results from the influence of prior extraction on the recycling decision. This dependence of recycling on extraction was pointed out by Judge Hand in the Alcoa case and debated at length in the economic literature.

Further calculations yield

$$(4.13) \quad W_r^1(q, r) = \delta [\eta P(s - q + r) + (1 - \eta) P'(s - q + r)(s - q)]$$

Observe that  $W_r^1(q, r) > 0$  ( $< 0$ ) when  $\eta = 1$  (0). When the extraction sector behaves as a social planner, welfare increases with the recycled quantity due to valuable stock extension, whereas the monopoly revenue of the extraction sector decreases with the recycled quantity because the market price decreases in the second period. From the social planner's standpoint, recycling expands the stock of the natural resource sold in the second period, which enhances the consumer surplus in the second period by  $P(\cdot)$  for all the units of resource the recycling company is selling. In contrast, from the monopolist's standpoint, recycling puts a downward pressure on the second-period market price, reflected by  $P'(\cdot)$ , which applies to  $s - q$ , i.e., all the units of the virgin resource left to be sold by the extraction sector. When  $\eta = 0$ , the recycling effect in (4.12) is negative, since  $\tilde{r}'(q)$  is upward sloping. Using the social planner's outcome as a

benchmark, one can argue that there is a tendency of a monopolist-extractor to extract “too little” of the resource prior to recycling.

Moving ahead on the analysis proves difficult at the level of generality used so far. We will work with functional specifications to solve explicitly for the equilibrium outcome. We will use the following framework with quadratic welfare functions:

Quadratic Framework (QF).

- $S(q) = aq - q^2/2$ , which yields the demand function  $P(q) = a - q$ ,
- $\delta = 1$ ,
- $s < 2a$ ,
- $s > a - c$ ,
- $a > c$ .

The three inequalities correspond respectively to (4.4), (4.7) and (4.9), using the quadratic specifications. Within QF, the extraction sector’s objective function is

(4.14)

$$W^1(q, r) = \eta q^2/2 + (a - q)q + \eta(a(s - q + r) - (s - q + r)^2/2 - (a - s + q - r)(s - q)) + (a - s + q - r)(s - q)$$

The recycling company’s objective function is

$$(4.15) \quad W^2(q, r) = a(s - q + r) - (s - q + r)^2/2 - cr - F,$$

which yields

$$(4.16) \quad R(q) = \begin{cases} a - c - s + q & \text{when } \underline{q} \leq q \leq \tilde{q}, \\ 0 & \text{otherwise,} \end{cases}$$

where  $\underline{q} = s + c - a$ . Substituting  $\tilde{r}(q)$  into (4.15), we obtain the reduced-form function

$$(4.17) \quad \mathbf{W}^2(q) = \frac{(a - c)^2}{2} + c(s - q) - F.$$

The solution of equation  $\mathbf{W}^2(q) = 0$  yields the maximum level of prior extraction that accommodates the recycling company

$$(4.18) \quad \tilde{q} = \min \left\{ s, \frac{(a-c)^2 + 2cs - 2F}{2c} \right\} .$$

It follows that the minimum threshold of fixed cost above which the market is not attractive enough to the recycling company is

$$(4.19) \quad \bar{F} = \frac{a^2 - c^2}{2} .$$

More precisely, if the fixed cost  $F$  is weakly lower than  $\bar{F}$ , then  $\underline{q} \leq \tilde{q}$  and there exists  $q$  inside  $[\underline{q}, \tilde{q}]$ , at which recycling can be accommodated. Otherwise, the recycling company stays out for all  $q \in [0, s]$ .

#### 4.4. The first-best equilibrium ( $\eta = 1$ )

In this section, we characterize the equilibrium outcome in which a social planner takes into account the consumer surplus derived both from virgin resource extraction and the recycled product sales. Anticipating the recycling company's reaction (4.11), the social planner chooses  $q$  in the first period to maximize

$$(4.20) \quad \mathbf{W}^1(q) = S(q) + \delta S(s - q + R(q)) .$$

This function is discontinuous at  $\tilde{q}$ , with a downward jump since  $W_r^1(q, r) > 0$  from (4.13). Let  $q_e^a$  denote the optimal extraction that accommodates the recycling company in the social planner's outcome, subscript  $e$  meaning *efficient* from the social standpoint. The first-order condition at the local maximum  $q_e^a$  is

$$(4.21) \quad P(q_e^a) - \delta P(s - q_e^a + \tilde{r}(q_e^a)) = -\delta P(s - q_e^a + \tilde{r}(q_e^a)) \tilde{r}'(q_e^a) .$$

As previously seen, the welfare effect of prior extraction can be decomposed into the balance effect, captured by the left-hand side of (4.21), and the indirect welfare effect due to recycling, reflected by the right-hand side of (4.21). Condition (4.21) can be interpreted as a variant of the “Hotelling rule” for a non-renewable and recyclable resource. Indeed, this condition tells us that the extraction sector must be indifferent between selling a unit of resource today or tomorrow, given that the tomorrow resource is both extracted and recycled. As the natural stock size  $s$  is increased by the recycled amount  $\tilde{r}(q_e^a)$  in the second period, the value  $P(q_e^a)$  of a unit of resource extracted in the first period must be the same as the present value  $\delta P(s - q_e^a + \tilde{r}(q_e^a))$  of a unit of resource sold in the second period, corrected by the recycling effect  $\delta P(s - q_e^a + \tilde{r}(q_e^a))\tilde{r}'(q_e^a)$ . Clearly, this is the spirit of the Hotelling rule. As  $\tilde{r}(q)$  is upward sloping, the second-period welfare is improved by  $P(\cdot)\tilde{r}'(\cdot)$  because recycling creates a valuable extension of the resource stock. Moreover, we are able to compare  $q_e^a$  with the efficient level  $q_0^e$  in the absence of recycling. From (4.3), we know that  $P(q_0^e) = \delta P(s - q_0^e)$ , and furthermore  $W_r^1(q, r) |_{(q=q_0^e, r=0)} = \delta [P(s - q_0^e)]$ . Assumption (4.4) implies that  $W_r^1(q, r) |_{(q=q_0^e, r=0)} > 0$ . Hence, the recycling possibility increases prior extraction at the first-best equilibrium. Moreover, the Hotelling rule is disrupted in that the resource price is increasing faster than the interest rate since

$$(4.22) \quad \frac{P(s - q_e^a + \tilde{r}(q_e^a))}{P(q_e^a)} > \frac{1}{\delta}.$$

**Proposition 15.** *Under assumptions (4.4), (4.7) and (4.9), the prospect of recycling increases the first-best level of prior extraction so that the resource price is rising more rapidly than the interest rate.*

To obtain further insight into the existence and social desirability of recycling, we now turn to the specification within QF. We have previously seen that the recycling

company enters the market for all  $q$  inside  $[\underline{q}, \tilde{q})$  provided that  $F < \bar{F}$ . Furthermore, solving (4.21) for  $q_e^a$  yields

$$(4.23) \quad q_e^a = \min \{a, s\}.$$

Hence, when the resource is scarce ( $s < a$ ), the best accommodation choice from the social planner's standpoint is to deplete the whole stock in the first period.

As (4.4) within QF requires  $s < 2a$ , we have  $q_e^a > q_0^e$ , where  $q_0^e = \frac{s}{2}$  is the optimal prior level of extraction when the extraction sector ignores the recycling possibility. Moreover, one can check that  $q_e^a > \underline{q}$ . Assuming now that  $F < \bar{F}$ , the extraction sector anticipates the recycling reaction (4.16) and chooses  $q$  to maximize

$$(4.24) \quad \mathbf{W}^1(q) = \begin{cases} \frac{1}{2}(a^2 - c^2 + 2aq - q^2) & \text{if } \underline{q} \leq q \leq \tilde{q}, \\ as - s^2/2 + sq - q^2 & \text{otherwise.} \end{cases}$$

This function is piecewise concave and discontinuous with a downward jump at  $\tilde{q}$ . If  $\tilde{q} \leq q_0^e$ , then  $\mathbf{W}^1(q)$  is increasing on  $[\underline{q}, \tilde{q}]$  because  $q_e^a > q_0^e$ , and thus  $W^1(q)$  achieves two local maxima at  $\tilde{q}$  and  $q_0^e$ . In that case, accommodating the recycling company cannot be an option. However, the social planner may choose to "promote" recycling by extracting  $\tilde{q}$  below  $q_e^a$  in the first period, in order to generate consumer surplus in the second period. As  $P(\tilde{q})$  exceeds the price  $P(q_e^a)$  that would blockade the recycling company's entry, residual demand in the second period results from  $P(s - \tilde{q} + \tilde{r}(\tilde{q}))$ , which raises the recycling company's welfare up to the minimum level that allows entry. If  $q_0^e \leq \tilde{q}$ , then  $\mathbf{W}^1(q)$  achieves two local maxima at  $\min\{\tilde{q}, q_e^a\}$  and  $q_0^e$ . Any change that lowers  $\tilde{q}$  can be said to make recycling more difficult: if initially the entry of the recycling company is accommodated at  $q_e^a$ , it moves closer to being promoted at  $\tilde{q}$ , which

occurs when  $\tilde{q} \leq q_e^a$ . From (4.18), an increase in the fixed cost for the recycling company reduces  $\tilde{q}$  below  $s$ , while leaving  $q_e^a$  unaltered, thus making entry more difficult. We can distinguish two cases depending on the resource abundance.

(i) : The resource is scarce ( $s < a$ ) so that the extraction sector commits to depleting the whole stock in the first period, i. e.,  $q_e^a = s$ , when the recycling company is accommodated. This commitment is possible only if  $q_e^a \leq \tilde{q}$ . From (4.18), this latter inequality holds when  $F$  falls below the minimum fixed cost for recycling to be promoted, i. e.,

$$(4.25) \quad F_s = \frac{(a - c)^2}{2}.$$

As  $a > c$  from (4.9), we have  $F_s < \bar{F}$ .

(ii) : The resource is relatively abundant ( $a \leq s$ ). The extraction sector can commit to accommodating the recycling company only if  $q_e^a \leq \tilde{q}$ , which holds when  $F$  falls below  $\min \{F_a, \bar{F}\}$ <sup>10</sup>, where

$$(4.26) \quad F_a = \frac{(a - c)^2}{2} + c(s - a).$$

Figure 4.1 reproduces the relevant aspects of the case where  $q_e^a = a$  and  $F \leq F_a$ , so that  $q_e^a \leq \tilde{q}$ . The curves are drawn using QF in the case where  $\eta = \beta = 1$ . They show how to find a unique geometric solution corresponding to the first-best equilibrium. The figure depicts the extraction sector's isowelfare curves and the reaction functions of both the extraction sector and the recycling company in  $(q, r)$  space. The dotted line  $GH$  represents the extraction sector's reaction function. This function cuts each of the extraction sector's isowelfare curves at its maximum. In particular, given  $r = 0$ ,  $W^1$  is maximized at the point  $G$  which coordinates are  $(q_0^e, 0)$ , with  $q_0^e$  equal to  $\frac{s}{2}$  within QF. Holding  $q_0^e$  fixed, the extraction sector does better when  $r$  is higher

<sup>10</sup>It turns out that  $F_a \leq \bar{F}$  only if  $s \leq 2a - c$ .

because  $W_r^1(q, r) |_{(q=q_0^e, r=0)} = a - \frac{s}{2} > 0$ . Thus, higher isowelfare curves represent higher welfare levels for the extraction sector. The recycling reaction function  $R(q)$  is made up of the three segments  $[A, B]$ ,  $[B, C]$  and  $[D, S]$ , where  $B, C, D$  and  $S$  have respective coordinates  $(\underline{q}, 0)$ ,  $(\tilde{q}, \tilde{r}(\tilde{q}))$ ,  $(\tilde{q}, 0)$  and  $(s, 0)$ . The isowelfare curve tangent to  $[B, C]$  at  $M$  meets the  $q$ -axis at  $E$ . Output  $q_e^a$  is vertically below  $M$  on the  $q$ -axis. The isowelfare curve passing through  $G$  intersects  $[B, C]$  at the point which coordinates are  $(q_e^i, \tilde{r}(q_e^i))$ , where

$$(4.27) \quad q_e^i = a - \frac{\sqrt{2(2a - s + c\sqrt{2})(2a - s - c\sqrt{2})}}{2}$$

provided that  $s \leq 2a - c\sqrt{2}$ . Figure 4.1 illustrates the case where  $\underline{q} \leq q_0^e$ , which happens only if  $s \leq 2(a - c)$ , and so  $q_e^i$  actually exists.

Assume for a moment that  $s \leq 2(a - c)$ . When  $\tilde{q}$  is lower than  $s$ , the expression (4.18) shows that the position of the point  $D$  depends on  $c, F, s$  and  $a$ . The comparison of  $q_e^a$  and  $\tilde{q}$  determines whether the extraction sector accommodates or promotes recycling in equilibrium. In Figure 4.1, the extraction sector does better than the point  $C$  by setting prior extraction at  $M$ , so that recycling is accommodated. From the case where  $q_e^a \leq \tilde{q}$ , an increase in the fixed cost for the recycling company lowers  $\tilde{q}$  while leaving  $q_e^a$  unaltered, thus making entry more difficult. If the fixed cost of recycling is large enough that  $D$  lies to the right of  $I$  which coordinates are  $(q_e^i, 0)$  and to the left of  $E$ — which occurs in two cases: first, when  $s \leq a$  and  $F_s \leq F$ ; and second, when  $a \leq s \leq 2(a - c)$  and  $F_a \leq F$  —, the extraction sector prefers extracting  $\tilde{q}$  over  $q_e^a$  to make entry worthwhile for the recycling company: the first-best requires to promote recycling.<sup>11</sup>

<sup>11</sup>If prior extraction is set equal to  $\tilde{q}$ , the recycling company is actually indifferent between staying out and entering to yield the point  $C$ . However, its entry would increase the extraction sector's welfare substantially. Therefore, so long as the social planner thinks that there is a positive probability of entry with  $\tilde{q}$ , there is a discontinuous upward jump in the expected welfare from  $D$  to  $C$ . We adopt the convention here that the recycling company chooses to enter the market when it is indifferent.



Finally, if the fixed cost of recycling is so high that  $D$  lies to the left of  $I$ , the extraction sector blockades the entry of the recycling company with  $q_0^e$ , i. e., the same level of prior extraction as that prevailing in the absence of recycling possibilities. Hence, the extraction sector ignores recycling when  $\tilde{q}$  falls short of  $q_e^i$ , which is tantamount to  $F_i \leq F < \bar{F}$ , where

$$(4.28) \quad F_i = F_a + \frac{c\sqrt{2(2a-s+c\sqrt{2})}(2a-s-c\sqrt{2})}{2}$$

is the minimum fixed cost for recycling to be ignored. Further calculations show that  $F_i \leq \bar{F}$  when  $s \leq 2(a-c)$ .

Let us now turn to the case where  $s > 2(a-c)$  so that  $q_0^e < \underline{q}$ . Then,  $q_e^i$  does not exist for all  $s$  inside  $(2(a-c), 2a-c\sqrt{2})$  because  $W^1(q_e^a, \tilde{r}(q_e^a))$  is strictly lower than  $W^1(q_0^e, 0)$ : hence, recycling cannot be accommodated in equilibrium. Clearly, in that case we also have that  $\tilde{q} > q_0^e$ , and thus the first-best requires to ignore recycling.

The next proposition summarizes this discussion.

**Proposition 16.** *Under assumptions (4.4), (4.7) and (4.9) within QF, the first-best solution requires*

(1) *to accommodate the recycling company*

- *with  $q_e^a = s$  when  $a-c < s \leq \min\{a, 2(a-c)\}$  and  $F \leq F_s$ ,*
- *with  $q_e^a = a$  when  $a \leq s \leq 2(a-c)$  and  $F \leq F_a$ ;*

(2) *to promote recycling with  $\tilde{q}$*

- *when  $a-c < s \leq \min\{a, 2(a-c)\}$  and  $F_s < F < F_i$ ,*
- *when  $a \leq s \leq 2(a-c)$  and  $F_a < F < F_i$ ;*

(3) *to ignore recycling with  $q_0^e$  otherwise.*

The first-best solution requires to ignore recycling when the resource is abundant ( $s > 2(a - c)$ ) or when it is scarcer but the fixed cost of recycling is too high ( $F \geq F_i$ ). Hence, recycling is socially desirable only if the resource is sufficiently scarce and the fixed cost sufficiently low. Under these circumstances, the extraction sector either accommodates or promotes recycling. Recycling is accommodated when the entry of the recycling company is taken for granted. In that case, the first-best requires the extraction sector to set prior extraction above the level prevailing with no possibility of recycling: the consequent extension of the resource stock generates consumer surplus via recycled material. When the resource is significantly scarce ( $s \leq a$ ), the social planner commits to depleting the whole resource stock in the first period in order to accommodate recycling.

However, the social planner cannot take entry for granted when the fixed cost has intermediate values ( $F_s < F < F_i$  or  $F_a < F < F_i$ ). Instead of accommodating recycling, the extraction sector must promote recycling by reducing prior extraction in a way that generates sufficient consumer surplus for the recycling company to enter. Whether the social planner accommodates or promotes recycling, the possibility of recycling increases prior extraction relative to what would be optimally extracted with no possibility of recycling.

#### 4.5. A monopolist in the resource extraction sector ( $\eta = 0$ )

In this section, we focus on the situation in which a monopolist in the resource extraction sector is confronted by an independent competitive company in the recycling industry. This will provide a useful comparison with the monopoly analysis of a non-recyclable exhaustible resource in Stiglitz (1976). We substitute  $\eta = 0$  into (4.1) to get the extraction monopolist's objective function. Bearing in mind the recycling reaction

(4.11), the monopolist chooses  $q$  to maximize

$$(4.29) \quad \mathbf{W}^1(q) = P(q)q + \delta [P(s - q + R(q))(s - q)].$$

The function  $\mathbf{W}^1(q)$  is discontinuous at  $\tilde{q}$ , where there is an upward jump since  $W_r^1(q, r) < 0$  (see (4.13)). Let  $q_m^a$  denote the local maximum that accommodates recycling in the monopolist's outcome. The first-order condition at  $q_m^a$  is given by

$$(4.30) \quad \begin{aligned} & P(q_m^a) + P'(q_m^a)q_m^a - \delta P(s - q_m^a + \tilde{r}(q_m^a)) + P'(s - q_m^a + \tilde{r}(q_m^a))(s - q_m^a) \\ &= -\delta P'(s - q_m^a + \tilde{r}(q_m^a))\tilde{r}'(q_m^a)(s - q_m^a). \end{aligned}$$

Condition (4.30) has a familiar interpretation in the economics of exhaustible resources (see Stiglitz, 1976). The extraction monopolist compares the marginal revenue today with the discounted marginal revenue obtainable by postponing the extraction until tomorrow. The difference here from the previous literature is that recycling the resource both augments the stock size and gives rise to a perfect substitute for further quantities of the extracted resource. The left-hand side of (4.30) measures the aforementioned balance effect of prior extraction on the expected revenue in both periods, given that the available stock is  $s + \tilde{r}(q_m^a)$ . Were this effect set equal to zero, it would correspond to the Hotelling rule in the case investigated by Stiglitz (1976), where the monopoly power is unrestrained by recycling. Bearing in mind the possibility of recycling, the monopolist strategically anticipates the impact of prior extraction on the interaction between recycling and further extraction. As previously shown, this strategic effect reduces the second period price, because  $R(q)$  is upward sloping: increasing prior extraction triggers a more aggressive reaction by the recycling company, which decreases the second-period price by  $P'(\cdot)\tilde{r}'(\cdot)$ . The resulting downward pressure on price scales down the second-period marginal revenue from extraction. This provides the monopolist

with an incentive to look “friendly” from the start and extract less resources than the Hotelling rule would require in the absence of strategic effect. Such a strategy has the flavor of the so-called “puppy-dog” profile in the terminology of business strategies (see Fudenberg and Tirole, 1984). The extraction monopolist commits the recycling company to softening competition between recycling and further extraction. However, the “puppy-dog” strategy obeys here the inescapable logic of the extraction rule that the marginal revenue must rise at the rate of interest.

**Proposition 17.** *Under assumptions (4.4), (4.7) and (4.9), the prospect of recycling reduces the level of prior extraction set by the monopolist to accommodate recycling.*

The monopolist may like the possibility of preventing rather than accommodating recycling. We examine now entry conditions using the specification within QF. We substitute  $\eta = 0$  into (4.14) and write the extraction monopolist’s objective function as

$$(4.31) \quad W^1(q, r) = (a - q)q + (a - s + q - r)(s - q).$$

Moreover, solving (4.30) for  $q_m^a$  within QF, we explicitly compute the optimal extraction that accommodates recycling as

$$(4.32) \quad q_m^a = \frac{a - c}{2},$$

provided that  $q_m^a > \underline{q}$ , or, equivalently,  $s < \frac{3}{2}(a - c)$ : hence, the monopolist will not accommodate the recycling company if the resource is too abundant. Note that  $q_m^a < q_0^m$ , where  $q_0^m = \frac{s}{2}$  is the monopolist’s optimal output in the absence of recycling possibilities. This is consistent with the result stated in proposition 17 that the prospect of recycling induces the monopolist to extract strategically little in the first period. Anticipating the

recycling reaction (4.16), the monopolist chooses  $q$  to maximize

$$(4.33) \quad \mathbf{W}^1(q) = \begin{cases} (a - q)q + c(s - q) & \text{if } \underline{q} \leq q \leq \tilde{q}, \\ (a - q)q + (a - s + q)(s - q) & \text{otherwise.} \end{cases}$$

This function is piecewise concave and discontinuous with an upward jump at  $\tilde{q}$ . Assume first that the resource is so abundant that  $s \geq \frac{3}{2}(a - c)$ . Then,  $q_m^a \leq \underline{q}$  and  $\mathbf{W}^1(q)$  is decreasing on  $[\underline{q}, \tilde{q}]$ , thereby achieving a maximum at  $\max\{q_0^m, \tilde{q}\}$ . Straightforward calculations show that  $q_0^m \leq \tilde{q}$  for all  $F \leq F_i^m$ , where

$$(4.34) \quad F_i^m = \frac{(a - c)^2 + cs}{2}$$

is the minimum fixed cost for recycling to be ignored. In that case, the monopolist prefers extracting  $\tilde{q}$  over  $q_0^m$  to prevent recycling: this is a deterring strategy in the sense that the monopolist increases prior extraction above the level  $q_0^m$  that would be optimally extracted with no recycling. Note that  $q_0^m$  accommodates recycling in the present case. Increasing extraction up to  $\tilde{q}$  strengthens competition in the second-period, thereby reducing the second-period price down to the threshold at which the recycling company is not entering..

If the fixed cost exceeds the threshold  $F_i^m$ , then  $\tilde{q} < q_0^m$  and the best choice for the monopolist is to ignore recycling and exercise unrestrained monopoly with  $q_0^m$ .

Assume now that the resource is scarce so that  $s < \frac{3}{2}(a - c)$ , which amounts to  $\underline{q} < q_m^a$ . Then,  $\mathbf{W}^1(q)$  has two local maxima whenever  $q_m^a < \tilde{q}$ . Figure 4.2 shows how to find the unique geometric solution to this problem. The figure depicts the monopolist's isorevenue curves given by (4.31) and the recycling reaction (4.16) in  $(q, r)$  space. Given  $r = 0$ ,  $W^1$  is maximized at the point  $G$  which coordinates are  $(q_0^m, 0)$ . Holding  $q_0^m$

fixed, the extraction sector does worse when  $r$  is higher since  $W_r^1(q, r) < 0$ . Thus, lower isowelfare curves represent higher welfare levels for the monopolist. The isorevenue curve is tangent to  $[B, C]$  at  $M$ , and this curve meets the  $q$ -axis at  $E$  which coordinates are  $(\bar{q}, 0)$ . Figure 4.2 illustrates the case where  $D$  lies between  $E$  which coordinates are  $(\bar{q}, 0)$  and  $G$  which coordinates are  $(q_0^m, 0)$ . In that case, the monopolist does better than the point  $M$  by setting prior extraction at  $\tilde{q}$ , so that the recycling company stays out.<sup>12</sup> Again, this is a deterring strategy that pushes prior extraction above  $q_0^m$ .

Deriving the explicit formula for  $\bar{q}$  within QF, we get<sup>13</sup>

$$(4.35) \quad \bar{q} = \frac{s}{2} + \sqrt{2 \left( s(2 + \sqrt{2}) + c - a \right) \left( a - c - s(2 - \sqrt{2}) \right)}.$$

Further calculations show that  $\tilde{q} \leq \bar{q}$ , as depicted in Figure 4.2, holds only if  $F \geq F_a^m$ , where

$$(4.36) \quad F_a^m = \frac{(a - c)^2}{2} + c(s - a) - c \frac{\sqrt{2 \left( s(2 + \sqrt{2}) + c - a \right) \left( a - c - s(2 - \sqrt{2}) \right)}}{4}$$

thus corresponds to the maximum fixed cost below which the monopolist accommodates the recycling company. Indeed, for all  $F < F_a^m$ , the point  $D$  lies to the right of  $E$ , in which case the monopolist finds it worthwhile to accommodate the recycling company with  $q_m^a$ . In contrast, if the fixed cost is so high that  $D$  lies to the left of  $G$ , which is tantamount to  $F > F_i^m$ <sup>14</sup>, the best choice for the monopolist is to ignore the recycling company, thereby blockading entry with  $q_0^m$ .

We can summarize our results as follows.

<sup>12</sup>If prior extraction is set actually equal to  $\tilde{q}$ , the recycling company is indifferent between staying out and entering to yield the point  $C$ . However, its entry would increase the extraction sector's profit substantially. Therefore, so long as the monopolist thinks that there is a positive probability of staying out with  $\tilde{q}$ , there is a discontinuous upward jump in the expected profit from  $C$  to  $D$ . We adopt the convention here that the recycling company chooses to stay out when it is indifferent.

<sup>13</sup>One can easily check that  $a - c - s(2 - \sqrt{2}) > 0$  for all  $s < \frac{3}{2}(a - c)$ .

<sup>14</sup>Further calculations show, first, that  $F_a^m < F_i^m$  for all  $s < \frac{3}{2}(a - c)$ , and second, that  $F_i^m < \bar{F}$  for all  $s < 2(a - c)$ .

**Proposition 18.** *Under assumptions (4.4), (4.7) and (4.9) within QF, the best choice for the monopolist is:*

(1) *to accommodate recycling with  $q_m^a$  when  $a - c < s < \frac{3}{2}(a - c)$  and  $F < F_a^m$ ,*

(2) *to deter recycling with  $\tilde{q}$*

- *when  $a - c < s \leq 2(a - c)$  and  $F \in [F_a^m, F_i^m)$ ,*

- *or when  $2(a - c) < s \leq 2a$ ;*

(3) *to ignore recycling with  $q_0^m$  when  $\frac{3}{2}(a - c) \leq s < 2(a - c)$  and  $F \in [F_i^m, \bar{F}]$ .*

The monopolist accommodates recycling only if the resource is scarce ( $s < \frac{3}{2}(a - c)$ ) and the fixed cost of recycling falls below the threshold  $F_a^m$ . In that case, the monopolist extracts strategically little in the first period—actually less than what would be extracted with no recycling—to soften competition between recycling and further extraction. If the fixed cost exceeds  $F_a^m$  when the resource is scarce, the monopolist finds it more profitable to prevent the entry of the recycling company, but cannot behave as if recycling were irrelevant. Recycling is actually seen as a threat by the monopolist which reacts by implementing the following deterrence strategy: the monopolist raises prior extraction above the level prevailing with no recycling in order to reduce the second-period price down to the threshold at which the recycling company is staying out. Recycling deterrence is also the monopolist's best strategy when the resource is abundant ( $s > 2(a - c)$ ), or moderately abundant ( $\frac{3}{2}(a - c) \leq s < 2(a - c)$ ) and the fixed cost is not too high ( $F \in [F_a^m, F_i^m)$ ). For higher values of the fixed cost in the case where the resource is moderately abundant, the monopolist can ignore recycling and its best strategy is to behave as if there were no threat of entry.

We finally compare the monopolist's optimal behavior to the first-best outcome. Table 1 summarizes the findings within QF stated in Propositions 16 and 18.

**Table 1****First-best outcome versus  
Monopolist's outcome****Scarce resource**

$$a - c < s < \frac{3}{2}(a - c)$$

First-best choice

$$s \leq a$$

accommodate recycling with  $q_e^a = s$  for all  $F \leq F_s$ promote recycling with  $\tilde{q}$  for all  $F \in (F_s, F_i)$ ignore recycling with  $q_0^e$  for all  $F \in [F_i, \bar{F}]$ 

$$s \geq a$$

accommodate recycling with  $q_e^a = a$  for all  $F \leq F_a$ promote recycling with  $\tilde{q}$  for all  $F \in (F_a, F_i)$ ignore recycling with  $q_0^e$  for all  $F \in [F_i, \bar{F}]$ 

Monopolist's choice

accommodate recycling with  $q_m^a = \frac{a-c}{2}$  for all  $F < F_a^m$ deter recycling with  $\tilde{q}$  for all  $F \in [F_a^m, F_i^m)$ ignore recycling with  $q_0^m$  for all  $F \in [F_i^m, \bar{F}]$ **Moderately abundant**

$$\frac{3}{2}(a - c) \leq s \leq 2(a - c)$$

**resource**

First-best choice

$$s \leq a$$

accommodate recycling with  $q_e^a = s$  for all  $F \leq F_s$ promote recycling with  $\tilde{q}$  for all  $F \in (F_s, F_i)$ ignore recycling with  $q_0^e$  for all  $F \in [F_i, \bar{F}]$ 

$$s \geq a$$

accommodate recycling with  $q_e^a = a$  for all  $F \leq F_a$ promote recycling with  $\tilde{q}$  for all  $F \in (F_a, F_i)$ ignore recycling with  $q_0^e$  for all  $F \in [F_i, \bar{F}]$ 

Monopolist's choice

deter recycling with  $\tilde{q}$  for all  $F < F_i^m$ ignore recycling with  $q_0^m$  for all  $F \in [F_i^m, \bar{F}]$ **Abundant resource**

$$2(a - c) < s \leq 2a$$





( $F < F_a^m$ ). Although recycling is socially desirable in that case, the monopolist under-extracts the resource relative to the first-best extraction to soften further competition with the recycling company.

#### 4.6. Conclusion

In this chapter, we have examined the best extraction strategies for an exhaustible resource that is recycled by an independent competitive company. For this, we have determined the equilibria of a two-period entry model in which an extraction sector chooses its best strategy bearing in mind the reaction of the recycling company.

Our findings allow for the comparison of the first-best solution and the monopolist-extractor's outcome. As the extraction sector creates its own competition whenever recycling proves feasible, the extraction choice prior to recycling is of great strategic importance. Unlike a social planner, the monopolist views recycling as a threat rather than an opportunity.

We find that, depending on the underlying parameters, the monopolist implements three kinds of equilibrium strategies in the face of recycling: the monopolist ignores, deters or accommodates recycling for decreasing values of the recycling fixed costs and increased scarcity of the virgin resource. Recycling deterrence departs from the strategy of ignoring recycling in that the monopolist increases prior extraction with the aim of pushing the future price of the resource down enough to make the market unattractive to potential recyclers. The monopolist may ignore or deter recycling under circumstances where, given the resource scarcity and the fixed costs magnitudes, the first-best requires to accommodate or promote recycling. When the resource is significantly scarce, it may happen that the monopolist accommodates recycling. This strategy implies extracting little to soften future competition against recycling, while, on the contrary, the first best requires to increase prior extraction when the recycling company is present.

The one-shot model with sequential moves takes only a limited account of the dynamics inherent in the problem of recycling an exhaustible resource. An improved treatment would be to switch to an infinite-horizon model from the two-period model.

In the present chapter, like in the previous chapters, we have disregarded the polluting nature of extraction and the green nature of recycling. In the following chapter, we will take this issue into account.

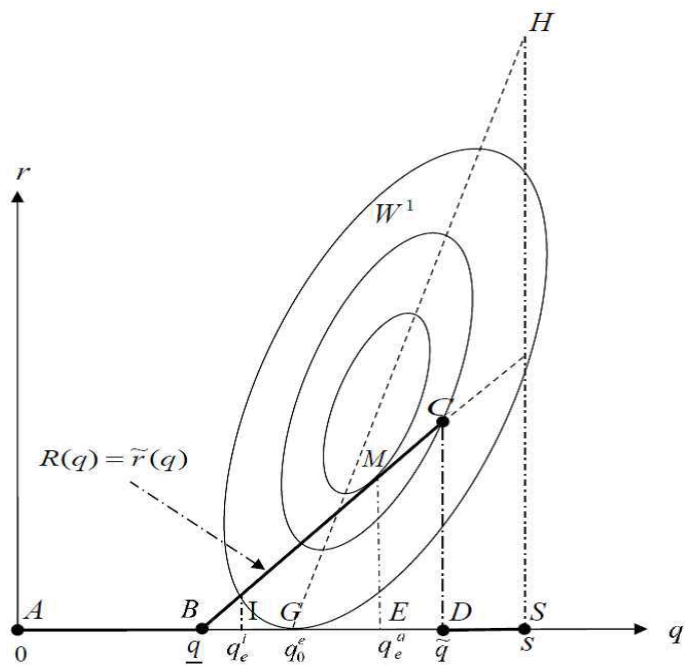


Figure 4.1. First-best solution

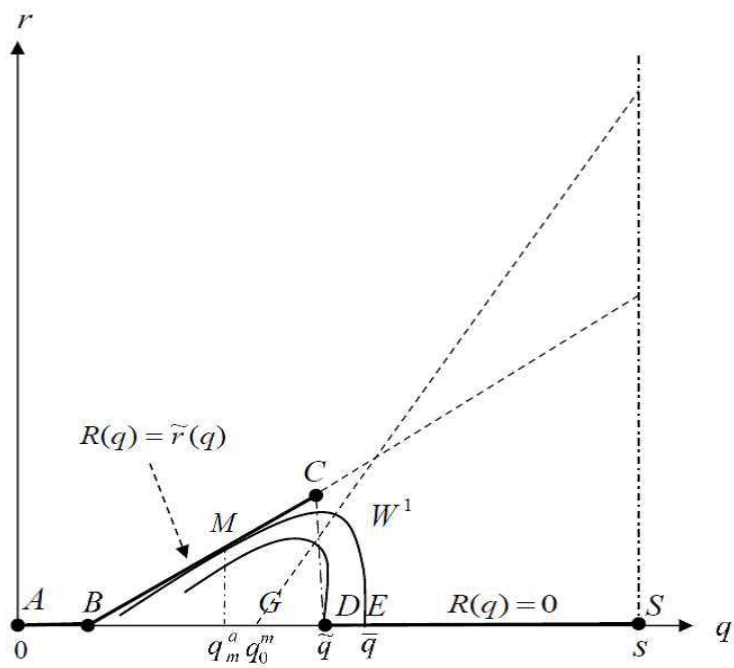


Figure 4.2. Monopolist's solution

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## CHAPTER 5

## Phosphorus Conservation and Reduction of Water Pollution: Tax and Subsidy Combination

### Abstract

This chapter analyzes the role of an environmental tax-subsidy scheme as an instrument for preserving phosphate reserves and for improving water quality by reducing water pollution. Toward these goals, we use a model where one firm (that can behave as a price-taker or as a price-maker) extracts and recycles phosphorus. We assume the presence of a benevolent government that regulates the market by taxing extracted phosphorus and subsidizing recycled phosphorus. First, we show that taxing extracted phosphorus and subsidizing recycled phosphorus contribute to the postponement of the depletion of the resource and to the reduction of pollution. Second, we show that, in the case where the firm behaves as a price taker, only a Pigovian tax is necessary and it enables to achieve the first-best. Conversely, if the firm is a price-maker, the combination of the two policies is needed. Third, we show that the tax-subsidy scheme does not modify the overall production supplied by the producer. Fourth, we show that the structure of the market is determinant in the ways to set the rate of the subsidy.

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**Keywords:** Environmental Tax, Subsidy, Phosphorus, Eutrophication, Recycling.

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## 5.1. Introduction

The conservation of phosphorus and the reduction of water pollution caused by phosphorus are two problems that a policy maker may be confronted with. On the one hand, it is well known that phosphorus, which has no substitute in agriculture<sup>1</sup> (Cordell and White, 2011), will be depleted in a near future (Cordell<sup>2</sup> and al., 2009). On the other hand, it is recognized that phosphorus pollutes water by creating eutrophication<sup>3</sup> (Iho, 2010). While it is true that extracted phosphorus pollutes water, it is also acknowledged that recycled<sup>4</sup> phosphorus reduces pollution for, at least, two reasons. First, recycling<sup>5</sup> prevents phosphorus from ending up into water and limits therefore eutrophication. Second, recycling reduces the production of waste and improves therefore the quality of the environment (Weikard and Seyhan, 2009; Cordell and al., 2011; Cogoye, 2009; Beir and Girmens, 2009). Accordingly, extraction generates a negative externality, while recycling generates a positive externality. In connexion with both phosphorus issues known as "phosphorus paradox" (Jean-Marie, 2013), the challenge for a social planner is to find solutions which (i) delay the depletion of this resource, (ii) and which avoid or limit eutrophication. Potential solutions for reaching these two goals consist of taxing<sup>6</sup>

<sup>1</sup>Note that phosphorus is essential for agriculture in the sense that it increases agricultural yields.

<sup>2</sup>They highlight that phosphate reserves may be depleted in 50 – 100 years.

<sup>3</sup>Recall that the term "Eutrophication" refers to an unwanted explosion of living aquatic-based organisms in lakes and estuaries that results in oxygen depletion, which can destroy an aquatic ecosystem (Liu and al., 2008). Significant eutrophication took place in the 1950s in the Great Lakes of North America, in Cayuga Lake, which is in Central New York (Jacobs and Casler, 1979), in the Poyang Lake watershed that is in China (Deng and al, 2011), in the Norfolk broads of United Kingdom (Philipps, 1984). It has been also prevalent in many other lakes and estuaries around the world (International Lake Environment Committee Foundation, 2003), and in the Baltic Sea.

<sup>4</sup>In addition to the reduction of eutrophication, recycling contributes to create energy and to save phosphorus. For instance, Li and al. (2015) argue that the implementation of a factory in China (in Chongqing town), which treats wastewater and sludge, allows to save 67,000 KWh per day and enables to generate 1 ton of phosphorus per day.

<sup>5</sup>It is noteworthy to mention that if recycled phosphorus ends up into water, it yields the same effect which is triggered by extracted phosphorus. However, in order to focus on the benefit of recycling in the reduction of eutrophication, we assume, in the world of this model, that recycled phosphorus does not end up into the water after its consumption.

<sup>6</sup>Since Pigou (1920), it is well known that negative externalities caused by pollution would be internalized by the market if polluters paid a tax equal to the marginal social cost of polluting emissions.

extracted phosphorus, subsidizing<sup>7</sup> recycled phosphorus, or combining both policy instruments. In order to clarify expectations, let us mention that this chapter deals with the last solution. We assume that both extracted and recycled products are supplied by one firm<sup>8</sup> that can behave as a price-taker or as a price-maker. When the firm behaves as a price-taker, the regulator only needs to use one instrument. However, when the products are supplied by a monopolist, two sources of misallocation can occur. One is the distortion due to the negative externality, and the other is the underproduction of final products generally associated with the exercise of the market power (Barnett, 1980). The co-existence of two distortions suggests that two policy instruments should be used simultaneously (David and Sinclair-Desgagné, 2010). Then, we will consider appropriate ways to combine a tax and a subsidy. Such a combination is what Fullerton and Wolverton (1999) call a two-part instrument (2PI). A practical example of a 2PI is a deposit-refund system<sup>9</sup> on items such as glass bottles or aluminum cans (Fullerton and Wolverton, 2003), and the best-known example of deposit-refund system is the system of fees and reimbursements for beverage containers<sup>10</sup> that has taken place in the United States (Walls, 2011). This system was originally adopted to combat litter problems and can be used to address many other environmental problems well beyond waste disposal, such as air and water pollution, in the same way as a Pigovian tax (Bohm, 1981). The system analyzed in this chapter is similar. In addition of reducing water pollution, the combination of taxation and subsidy postpones the extraction of the resource.

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<sup>7</sup>Several economists stress that it would be desirable to subsidize a polluter in order to induce him to abate pollution or to subsidize green products, which generate a positive externality.

<sup>8</sup>Note that this model is extended over two periods (For more details, see appendix II).

<sup>9</sup>A deposit-refund system combines a tax on product consumption with a rebate when the product or its packaging is returned for recycling or appropriate disposal (Walls, 2011). Equivalently, it consists of saying that the polluting product is taxed and recycling is subsidized.

<sup>10</sup>Deposit-refunds have been established for other kinds of containers such as, lead-acid batteries, motor oil, electronics (Walls, 2011).

Several countries have yet implemented phosphorus taxation, in order to reduce water pollution. These include the United States of America<sup>11</sup> (see Jacobs and Casler, 1979; Shakhramanyan and al., 2012), Sweden (Sjöberg, 2005) and China. Aiming at reducing pollution and saving phosphorus, some Swedish municipalities subsidize recycling (Kvarnström and Nilsson, 1999).

In the present chapter, we investigate the following main questions. Does the tax-subsidy scheme contribute to the prolongation of phosphorus lifetime and the reduction of water pollution? When is the first-best achieved with the implementation of this policy instrument?

In order to answer these questions, we postulate a model in which one firm (that can behave as a price-taker or as a price-maker) produces simultaneously a polluting resource and a clean resource. We assume the presence of a benevolent<sup>12</sup> government which sets simultaneously the level of the tax and that of the subsidy, and firm produces accordingly. We show our results using fairly general inverse demand, production costs, and damage functions.

Summarizing some of our findings, we can state the following main results. Firstly, a tax-subsidy scheme postpones the exhaustion of phosphorus and reduces water pollution. Secondly, we show that, in the case where the firm behaves as a price taker, only a Pigovian tax is necessary and it enables to achieve the first-best. Conversely, if the firm is a price-maker, the combination of the two policies is needed. In this case, the tax is lower than the marginal social damage. Thirdly, we outline that the structure of the market is determinant in the ways to set the rate of the subsidy.

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<sup>11</sup>Aiming at reducing eutrophication of Cayuga Lake (United States), Jacobs and Casler (1979) compare an effluent tax policy with a uniform reduction policy. They stress that it is less costly to reduce eutrophication by taxing effluents than by aiming at reducing it uniformly. For instance, they estimate that the reduction of phosphorus discharge at a level of 10 percent costs 32,065\$ in the case of the tax on effluents, whereas it costs 37,177\$ in the case of the uniform reduction policy.

<sup>12</sup>We prefer to keep the term "benevolent" in the sense that the government cares about the whole society, which consists here of the firm and consumers.

This chapter is based on several strands of the literature: the first one is related to the analysis of the effect of taxation on environmental pollution. Recalling that the Pigovian conclusion that the level of the tax must equal the marginal social cost of polluting emissions is made within a context of perfect competition. In fact, when the market is imperfectly competitive, the tax should be set lower<sup>13</sup> than the marginal social cost of pollution, because it trades off the desire to provide incentives for abatement and the necessity to prevent a greater contraction of output (Nimubona and Sinclair-Desgagné (2005), see also Levin (1985)). Buchanan (1969) echoes the previous conclusion and argues that setting a tax lower than the marginal social damage solves the tendency of imperfectly competitive firms to underproduce. The same conclusion is highlighted by Barnett (1980) in the case where the elasticity of demand is finite. Nevertheless, other authors show that the corrective tax can be higher than the marginal social cost. Using a Cournot duopoly model, Simpson (1995) finds that the optimal tax rate may exceed the marginal damage because Cournot duopolists will not, in general, produce their outputs efficiently. To the extent that a pollution tax may shift production output from the less efficient firm to its more efficient rival, higher tax rates may be needed. David and Sinclair-Desgagné (2005) state that, under some conditions, an optimal emission tax should be set higher than the marginal social cost of pollution. The intuition underlying this idea is that imperfect competition between environment firms results in abatement prices larger than the marginal social cost of abatement; emission taxes must then be raised in order to make polluters reduce their emissions sufficiently. Barnett (1980) finds that the tax can be higher than the marginal social damage if demand

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<sup>13</sup>Through our two-period model that a curious reader should find in appendix II, we show also it is possible that the tax applied to a monopoly be higher than the marginal social damage. This result is not in line with the conventional wisdom which states that the tax applied to a monopoly must be lower than the marginal damage of pollution. The goal is to prevent a greater contraction of output. Such a result is not usual. It has been shown by some authors but within a duopoly context. Within the context of a monopoly, to our knowledge, only Barnett (1980) has highlighted the possibility to find a similar result. His conclusion is valid only in the case where demand is more price elastic.

is more price elastic. Using a Bertrand duopoly model with differentiated products, Kurtyka and Mahenc (2011) show that the corrective tax can be higher than the marginal damage. The intuition behind their result is that high taxation of the polluting good will encourage consumers to switch toward the green product. In spite of the lack of consensus on the level of the tax with respect to the marginal social damage inflicted by the polluter, the overall conclusion of this line of research is that taxation reduces pollution. However, there are some situations where the effect of the environmental tax may induce an undesirable effect. Levin (1985) analyzes this issue within a Cournot oligopoly model. He assumes that a tax is imposed on each seller at a uniform rate per unit of output and shows if firms are sufficiently different, pollution increases in the tax rate. The intuition that he provides is that if firms are asymmetric and one taxes them symmetrically, there is a chance that the heavy polluter will benefit, resulting in the rise of pollution. Although our finding that the tax is lower than the marginal social damage is in line with the first line of research, our model differs from the other models, because it considers a monopolist which produces two types of products.

The second strand of the literature concerns the relationship between a subsidy and pollution control. Baumol and Oates (1995) argue that although a subsidy will tend to reduce the emission of the firm, it can increase the emissions of the industry. Mestelman (1982) uses a general equilibrium model and analyzes the effects of taxes and subsidies in a competitive economy which is characterized by a production externality. He shows that the use of a subsidy is inefficient, relative to the alternative of taxation. Mestelman (1984) shows that a pollution tax is consistently preferred to a subsidy by majority of individuals. Diamond and Mirrlees (1971) find that the efficient combination of abatement and output requires the Pigovian tax alone, because the pollution abatement subsidy

distorts the price of the input used for reducing pollution, resulting thus in an inefficiency. Fredriksson (1997) highlights that pollution abatement subsidies are inefficient instruments for pollution control. In contrast to Diamond and Mirrlees (1971), Mestelman (1982), and Fredriksson (1997), we show that the subsidy is efficient in pollution control in the sense that it leads to the reduction of pollution.

The remainder of the chapter is structured as follows. The next section provides some context as regards how phosphorus pollutes water. Section 5.3 presents the description of the model. Section 5.4 describes the first-best situation. The combination of taxation and subsidy is studied in section 5.5, whereas an illustrative example is exposed in section 5.6. Section 5.7 extends the basic model. The main conclusions and some further research lines are provided in section 5.8, all proofs and a two-period model are relegated to the appendix in sections 5.9 and 5.10.

## **5.2. Phosphorus and Water Pollution: Eutrophication**

Bodies of water can be categorized as being in one of two states on the basis of their nutrient content. Low nutrient oligotrophic water are clear and have relatively little animal and plant life, whereas the high nutrient content of eutrophic water encourages the development of fauna and flora (Salerno, 2009). Eutrophication denotes the enrichment in nutrients of lakes and rivers that leads to this state of abundant life and therefore sounds like a positive development for a natural habitat. This enrichment can disrupt the natural balance of the natural system and lead to a complete transformation of the habitat (Ricklefs, 1979). The new altered state is often characterized by rapid plant and algae growth. When the density of the vegetation becomes such that the ecosystem can no longer support it, it dies and begins to decay (Salerno, 2009). Since the rate of decomposition increases, the process consumes so much oxygen that fish and other aquatic animals suffocate (Ricklefs, 1979). In addition, the growth of non-toxic algae

results in shade and an rise of the water pH<sup>14</sup>, which then favors the abundance of the cyanobacteria or blue-green algae, a bacterium that can produce lethal toxins (Scheffer, 1998). Algae can also affect treatment of water for potable supply, by blocking filters or passing through them causing bad odeur and taste (Collingwood, 1977).

According to Ramade (1981), eutrophication manifests in four stages:

(i) Increasing pollution: phosphorus ends up into water, due to water run-off, soil erosion, etc. At the beginning, the oxygen content favors aquatic life. Fish are not affected.

(ii) Algae growth: phosphorus leads to the development of algae which consume so much oxygen. The oxygen content increases at the surface of the water but diminishes significantly in the depths of the water. Some species die.

(iii) Anaerobic decomposition: sediments rich in organic matter accumulate more. Aerobic bacteria multiply in order to degrade organic matter and consume oxygen. The oxygen content is strongly weakened on the whole water column.

(iv) Extreme degradation of the environment dystrophy stage : The oxygen content has significantly fallen. There is an absence of oxygen in the aquatic environment. The depletion of oxygen favors the formation of sulfuric acid and ammoniac in the water, leading to the death of fish. At this stage, there is a health risk for fauna and for humanity that use this water, because some cyanobacteria produce toxins.

### 5.3. The model

The economy we consider consists of a benevolent government, consumers and one firm that can behave as a price-taker or as a price-maker, and which extracts and recycles phosphorus simultaneously. Let  $q$  denote the quantity which is extracted by the firm,

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<sup>14</sup>The pH measures the acidity or the basicity of a solution. A solution with a pH of 7 is considered to be neutral. If  $pH < 7$ , the solution is considered to be acid and basic if  $pH > 7$  ([https://fr.wikipedia.org/wiki/Potentiel\\_hydrog%C3%A8ne](https://fr.wikipedia.org/wiki/Potentiel_hydrog%C3%A8ne)).

$c(q)$  the cost of production,  $r$  the quantity that it recycles,  $c(r)$  the cost of recycling and  $p(Q)$  the inverse demand function, where  $Q$  is the total quantity and is given by the sum of  $q$  and  $r$ . Extracted phosphorus pollutes water by ending up into rivers, oceans and lakes. Recycling phosphorus is a way of preventing phosphorus from ending up into waters. In addition, recycling contributes to waste management. Accordingly, each unit of extracted phosphorus generates a volume of polluting emissions given by  $e(q, r)$ . It is convenient to assume that the emission function takes the following form:  $e(q, r) = \varepsilon(q) - \delta(r)$ . It is reasonable to assume that  $\varepsilon'(q) > 0$ , because extraction increases emissions and  $\delta'(r) > 0$ , meaning that recycling decreases or limits<sup>15</sup> total emissions. We assume that the firm extracts and recycles phosphorus, simultaneously. The benevolent government taxes extracted phosphorus and subsidizes recycled phosphorus.

The timing of the game between the regulator and the producer can be described as follows. In the first stage, the regulator sets the level of the tax and that of the subsidy. In the second stage, the producer chooses the quantity it extracts and that it recycles.

In what follows, we will consider the first-best situation as a benchmark.

#### 5.4. The first-best

Extracting phosphorus entails an environmental damage  $d$ . The benevolent government maximizes the following social welfare function which is the difference between the sum of the firm's profit, i.e. the profit deriving from extraction of phosphorus and that deriving from recycling, consumer's surplus and any technological external effects which are not accounted for in firm's profits:

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<sup>15</sup>When one considers recycled phosphorus is sewage sludge (which can be seen by consumers as a waste) and the latter pollutes the environment, recycling can be seen as a mean of decreasing and limiting pollution. When this sewage sludge is not taken by consumers as a waste, recycling limits only eutrophication by preventing phosphorus from ending up into waters.



$$(5.1) \quad W(q, r) = \int_0^Q p(z)dz - c(q) - c(r) - d[\varepsilon(q) - \delta(r)],$$

where  $Q = q + r$ . First, assume that the firm behaves as a price taker. Then, the first-order conditions for welfare maximization are given by:

$$(5.2) \quad p - c'(q) - d\varepsilon'(q) = 0$$

$$(5.3) \quad p - c'(r) + d\delta'(r) = 0$$

The solution of the necessary conditions above yields what Park and al. (2012) call the principle of marginal optimality. Indeed, equation (5.2) indicates that the market price of phosphorus is equal to its marginal cost of production plus the marginal social damage inflicted by pollution. Equation (5.3), in turn, states that the market price of the resource is equal to the marginal cost of recycling less the marginal social benefit of recycling.

Notice that an unregulated firm will not a priori operate in a socially optimal way. For instance, in order to reach the social optimum, the regulator can adopt numerous environmental policies. In the case where the firm behaves as a price-maker, there are two distortions, i.e. the negative externality and the market power of the monopolist. In what follows, we will consider that the regulator applies a tax-subsidy scheme.

### 5.5. The tax-subsidy device

We first deal with the case of a price taking firm. Let  $\tau$  be the tax set by the regulator and  $s$  the level of the subsidy paid to the firm in order to boost recycling. The profit of

the firm is then given by:

$$(5.4) \quad \pi(q, r) = p \cdot (q + r) - c(q) - c(r) - \tau[\varepsilon(q) - \delta(r)] + sr$$

The first-order conditions for the representative firm's optimization problem are:

$$(5.5) \quad p - c'(q) - \tau\varepsilon'(q) = 0$$

$$(5.6) \quad p - c'(r) + \tau\delta'(r) + s = 0$$

The comparison of (5.2) with (5.5) and (5.3) with (5.6), enables us to conclude that the first-best can be achieved with a tax-subsidy scheme given by:

$$(5.7) \quad \tau = d$$

$$(5.8) \quad s = 0$$

The implications of the previous calculus are summed up through the next proposition.

**Proposition 19.** *When the firm acts as a price taker, the first-best is reached by applying only a Pigovian tax.*

This proposition indicates that the Pigovian tax is sufficient to achieve the first-best, because the regulator deals only with one distortion, i.e. the negative externality inflicted by pollution. The level of this tax internalizes fully the externality, because the tax set by the regulator corresponds to the marginal social damage.

Now assume that the firm is a monopolist. Then, the market price of the resource depends on the total output,  $Q$ , and equations (5.5) and (5.6) turn, respectively, into:

$$(5.9) \quad p(Q) - c'(q) - d\varepsilon'(q) = 0$$

$$(5.10) \quad p(Q) - c'(r) + d\delta'(r) = 0$$

The monopolist's profit is given by:

$$(5.11) \quad \pi(q, r) = p(Q)Q - c(q) - c(r) - \tau[\varepsilon(q) - \delta(r)] + sr$$

Then, the first-order conditions for the programme above yield:

$$(5.12) \quad p'(Q)Q + p(Q) - c'(q) - \tau\varepsilon'(q) = 0$$

$$(5.13) \quad p'(Q)Q + p(Q) - c'(r) + \tau\delta'(r) + s = 0$$

The comparison of (5.9) with (5.12), and (5.10) with (5.13) enables us to conclude that:

$$(5.14) \quad \tau = d + \frac{\overbrace{p'(Q)}^-}{\underbrace{\varepsilon'(q)}_+} Q$$

and

$$(5.15) \quad s = -\underbrace{p'(Q)}^- \cdot Q \left[ 1 + \frac{\overbrace{\delta'(r)}^+}{\underbrace{\varepsilon'(q)}_+} \right]$$

Equation (5.14) means that the tax is composed of the distortion from the negative externality and the distortion from the monopolist's market power per marginal emission. Since  $p'(Q) < 0$ , due to the fact that the price is a decreasing function of the total production, (5.14) implies that the tax is lower than the marginal social damage, i.e.  $\tau < d$ . Equation (5.15), in turn, states that the regulator sets a non-zero subsidy due to the co-existence of two distortions. From this equation, we know that higher is the distortion from the market power of the monopolist, higher is the amount of the subsidy. One may conclude then, the less competitive is the market, higher is the subsidy rate set by the regulator. Hence, the structure of the market is determinant in the ways to set the rate of the subsidy. From the calculus above, we establish the upcoming proposition.

**Proposition 20.** *When extracted and recycled products are supplied by a monopolist, the two instruments are necessary because there are two distortions—the negative externality inflicted by pollution and the market power of the firm. The tax charged by the regulator is below the marginal social cost of pollution.*

Proposition 20 can be explained as follows. If the firm behaves as a monopolist, the negative externality can be internalized with a tax rate set below the marginal social damage. The regulator sets the tax to this level in order to solve the tendency of the monopolist to underproduce.

In a nutshell, we notice that the case of a monopolist contrasts with the case of a price-taking firm. While the tax is sufficient to efficiently regulate the price-taking firm, it is not sufficient in the case of a monopolist. In this case, a tax-subsidy combination enables to reach efficiency.

In what follows, we will deal with specific functional forms in order to provide a clearer insight.

## 5.6. Specific example

Let us illustrate the general results by the following specific functions. Let  $p(Q) = a - q - r$ ;  $\varepsilon(q) = q$ ;  $\delta(r) = r$ ;  $c(q) = \frac{1}{2}q^2$  and  $c(r) = \frac{1}{2}r^2$ . Assume that the firm can behave either as a price taker, or as a monopolist. As in the general case, the first-best will constitute the benchmark situation.

### 5.6.1. The first-Best

Using the previous illustrative example and making some simplifications yield the social welfare function given by:

$$(5.16) \quad W(q, r) = \int_0^Q p(z)dz - \frac{1}{2}q^2 - \frac{1}{2}r^2 - d(q - r),$$

If the firm behaves as a price taker, the first-order conditions are given by:

$$(5.17) \quad p - q - d = 0$$

$$(5.18) \quad p - r + d = 0$$

From (5.17) and (5.18), we can establish the total quantity as follows:

$$(5.19) \quad Q^c = 2p^c,$$

where the superscript  $c$  refers to the term competition. After the implementation of the tax and the subsidy, the profit of the firm is:

$$(5.20) \quad \pi(q, r) = p.(q + r) - \frac{1}{2}q^2 - \frac{1}{2}r^2 - \tau(q - r) + sr$$

Its reply to this policy is then captured by the following profit-maximization first-order conditions:

$$(5.21) \quad p - q - \tau = 0$$

$$(5.22) \quad p - r + \tau + s = 0$$

Comparing (5.17) with (5.21) and (5.18) with (5.22) yields the upcoming proposition:

**Proposition 21.** *When the polluting good is supplied by a price-taking firm, the first-best is reached by using one instrument only, i.e. taxation of extracted phosphorus. Formally, we have:*

(i)

$$(5.23) \quad \tau = d$$

(ii)

$$(5.24) \quad s = 0$$

From (5.21), (5.22) and (5.24), the total quantity is given by:

$$(5.25) \quad Q^{c/\tau s} = 2p^c,$$

where  $Q^{c/\tau s}$  is the total production of the price-making firm after the implementation of the policy.

This examples confirms our general result that the level of the tax is equal to the marginal social damage in the case where the market is perfectly competitive. It shows also that using a tax only enables to achieve the first best in the sense that subsidy is

not needed here, i.e.  $s = 0$ . From this example, we also know that the implementation of the tax-subsidy scheme does not modify the level of the total quantity. In fact, taxation reduces extracted phosphorus on the one hand and increases recycled phosphorus on the other hand. But since the increasing effect is equal to the decreasing effect (one can observe it through (5.21) and (5.22)), the total production remains constant.

Now, let us consider that the firm behaves as a monopolist. The social welfare function is given by:

$$(5.26) \quad W(q, r) = a(q + r) - \frac{1}{2}(q + r)^2 - \frac{1}{2}q^2 - \frac{1}{2}r^2 - d(q - r)$$

The first-order conditions are given by:

$$(5.27) \quad a - 2q - r - d = 0$$

$$(5.28) \quad a - q - 2r + d = 0$$

The total quantity of the monopolist deriving from (5.27) and (5.28) is:

$$(5.29) \quad Q^m = 2p^m,$$

where  $p^m$  is the price charged by the monopoly and is given by  $p^m = a - q - r$ . The profit of the firm, after the implementation of a tax-subsidy scheme, is:

$$(5.30) \quad \pi(q, r) = (a - q - r)(q + r) - \frac{1}{2}q^2 - \frac{1}{2}r^2 - \tau(q - r) + sr$$

The first-order conditions are:

$$(5.31) \quad a - 3q - 2r - \tau = 0$$

$$(5.32) \quad a - 2q - 3r + s + \tau = 0$$

Solving fully the programme of the monopolist leads to the two following propositions.

**Proposition 22.** *Efficiency is achieved for a tax lower than the marginal social damage and a subsidy higher than the price of the resource. Formally, we have:*

(i)

$$(5.33) \quad \tau = d - \frac{2}{3}a$$

(ii)

$$(5.34) \quad s = \frac{4}{3}a$$

**Proof.** See appendix

□

Proposition 22 states that, in the presence of two distortions, the regulator must use two policy instruments. This will enable him to internalize the negative externality and to reduce the monopolist's tendency to underproduce.

As in the competitive case, the specific example corroborates our general result according to which the regulator sets a tax lower than the marginal social damage and a



positive subsidy rate, when the market is imperfectly competitive. From this example, we know that  $s = 4p^m$ , since  $p^m = \frac{1}{3}a$ .

**Proposition 23.** *The combination of both policies reduces water pollution and delays the depletion of the resource. Formally, we have:*

(i)

$$(5.35) \quad q(s, \tau) = \frac{a - 2s - 5\tau}{5}$$

(ii)

$$(5.36) \quad r(s, \tau) = \frac{a + 3s + 5\tau}{5}$$

**Proof.** See appendix □

Proposition 23 states that the combination of the two instruments reduces extraction and boosts recycling. Indeed, on the one hand, the imposition of the tax makes extracted phosphorus more expensive in that the extractor will set a higher price, which tends to lead consumers to switch towards recycled phosphorus which remains cheaper. On the other hand, subsidizing recycled phosphorus increases the revenues of the firm, resulting in the rise of the quantity it recycles. The combination of these two policies, in addition of the substitutability of these two products, contributes to the postponement of the extraction of phosphorus. Accordingly, the lifetime of the resource is prolonged and water pollution is reduced or limited.

From (5.31) and (5.32), the global production is given by:

$$(5.37) \quad Q^{m/\tau s} = \frac{2p^m + s}{3}$$

It is clearly shown through the appendix that:

$$(5.38) \quad Q^{m/\tau s} = Q^m,$$

where  $Q^{m/\tau s}$  is the total production of the monopolist after the implementation of the tax-subsidy scheme.

Thus, the implementation of the environmental policy has not modified the overall production.

### 5.7. Recycled phosphorus pollutes water

The basic model can be extended into two directions. The first one consists of introducing a sequentiality and assuming that recycled phosphorus does not end up into waters (see **appendix II**). The second one consists of assuming one fairly long period, which allows recycled phosphorus and extracted phosphorus to end up into waters, simultaneously. Consequently, both resources pollute water. In the present section, we follow the last direction. In the first-best situation, the social welfare function writes:

$$(5.39) \quad W(q, r) = \int_0^Q p(z) dz - c(q) - c(r) - d[\varepsilon(q + r) - \delta(r)]$$

First, assume that the producer behaves as a price taker. Then, the first-order necessary conditions for the maximization problem are as follows:

$$(5.40) \quad p - c'(q) - d\varepsilon'(q + r) = 0$$

$$(5.41) \quad p - c'(r) - d\varepsilon'(q + r) + d\delta'(r) = 0$$

After the implementation of the tax-subsidy scheme, the profit of the representative firm is:

$$(5.42) \quad \pi(q, r) = p \cdot (q + r) - c(q) - c(r) - \tau[\varepsilon(q + r) - \delta(r)] + sr$$

The first-order conditions for the firm's optimization problem are:

$$(5.43) \quad p - c'(q) - \tau\varepsilon'(q + r) = 0$$

$$(5.44) \quad p - c'(r) - \tau\varepsilon'(q + r) + \tau\delta'(r) + s = 0$$

Comparing (5.40) with (5.43), and (5.41) with (5.44) yields:

$$(5.45) \quad \tau = d$$

$$(5.46) \quad s = 0$$

Proposition 24 emphasizes this result.

**Proposition 24.** *When the market is competitive, implementing the tax only suffices to achieve the first-best.*

This result confirms the conventional wisdom that applying only a tax suffices to reach the first-best in a competitive market.

Let us now turn to the case where the two resources are supplied by a monopolist. Then, equations (5.40) and (5.41) become:

$$(5.47) \quad p(Q) - c'(q) - d\varepsilon'(q+r) = 0$$

$$(5.48) \quad p(Q) - c'(r) - d\varepsilon'(q+r) + d\delta'(r) = 0$$

After the implementation of the tax-subsidy device, the monopolist's profit is given by:

$$(5.49) \quad \pi(q, r) = p(Q)Q - c(q) - c(r) - \tau[\varepsilon(q+r) - \delta(r)] + sr$$

Then, the first-order conditions for the programme above yield:

$$(5.50) \quad p'(Q)Q + p(Q) - c'(q) - \tau\varepsilon'(q+r) = 0$$

$$(5.51) \quad p'(Q)Q + p(Q) - c'(r) - \tau\varepsilon'(q+r) + \tau\delta'(r) + s = 0$$

The comparison of (5.47) with (5.50), and (5.48) with (5.51) yields the following tax-subsidy scheme:

$$(5.52) \quad \tau = d + \frac{p'(Q)}{\varepsilon'(q+r)}Q$$

and

$$(5.53) \quad s = -p'(Q).Q \frac{\delta'(r)}{\varepsilon'(q+r)}$$

The discussions underlying these calculus will follow the next proposition.

**Proposition 25.** *When the market is imperfectly competitive (monopoly), the tax is lower than the marginal social cost and a positive subsidy rate is set by the regulator.*

The explanations behind proposition 25 are as follows. As in the previous lines, the tax has, here, two components: one distortion from the market power of the firm and another from the negative externality. It is set below the marginal social damage in order to reduce the monopolist's tendency to cut down the extraction.

### 5.8. Conclusion

This chapter analyzes the effect of the combination of a tax and a subsidy on the lifetime of phosphorus and the reduction of water pollution known as eutrophication. Considering one firm that can behave either as a price taker, or as a price maker, and that produces simultaneously a polluting resource and recycled products, and assuming the presence of a benevolent government that taxes pollution and subsidizes recycling, we state the following results. First, we show that taxing extracted phosphorus and subsidizing recycled phosphorus contribute to the postponement of the depletion of the resource and to the reduction of pollution. Second, we show that, in the case where the firm behaves as a price taker, only a Pigovian tax is necessary and it enables to achieve the first-best. Conversely, if the firm is a price-maker, the combination of the two policies is needed. Third, we show that the tax-subsidy scheme does not modify the overall production supplied by the producer. Fourth, we show that the structure of the market is determinant in the ways to set the rate of the subsidy.

For the sake of simplicity:

(i) We have not addressed the problem as the issue of an exhaustible resource through this model,

(ii) In the extension part, we have postulated a model which allows for the consideration of the polluting nature of recycled phosphorus, once it enters the water. Nevertheless, we have disregarded the sequential aspect. It would be interesting to set up a dynamic model which would substantially modify the social welfare function in the sense that it would add another damage in a subsequent period.

In order to take into account these two issues, we have implemented a two-period model which has confirmed our general results that one instrument is needed in the competitive case, whereas two instruments are necessary within an imperfectly competitive framework (see **appendix II**).

In the present chapter, we have assumed that both goods are supplied by one firm that can behave as a price taker or as a price maker. It would be interesting to imagine a case where the clean resource is provided by another industry. Then, instead of having an intrinsic competition between recycled phosphorus and extracted phosphorus, an extrinsic competition would prevail.

Another challenge for the future is to study the problem as a non-point source pollution issue, because phosphorus can pollute other areas due to water run-off and soil erosion.

### 5.9. Appendix I

**Proof of propositions 22 and 23:** Assume that the firm has a monopoly behaviour. Then, social welfare is given by:

$$(5.54) \quad W(q, r) = \int_0^Q p(z) dz - c(q) - c(r) - d[\varepsilon(q) - \delta(r)]$$

Under the specific example, the first-order conditions are:

$$(5.55) \quad a - 2q - r - d = 0$$

$$(5.56) \quad a - q - 2r + d = 0$$

The total quantity deriving from these two first-order conditions is:

$$(5.57) \quad Q^m = q + r = 2p^m,$$

where

$$(5.58) \quad p^m = a - q - r$$

After the implementation of the tax-subsidy scheme, the profit of the monopolist writes:

$$(5.59) \quad \pi(q, r) = (a - q - r)(q + r) - \frac{1}{2}q^2 - \frac{1}{2}r^2 - \tau(q - r) + sr$$

The first-order conditions are:

$$(5.60) \quad a - 3q - 2r - \tau = 0$$

$$(5.61) \quad a - 2q - 3r + s + \tau = 0$$

Form (5.60) and (5.61), one can obtain the optimal quantities given by:

$$(5.62) \quad q(s, \tau) = \frac{1}{5}(a - 2s - 5\tau)$$

$$(5.63) \quad r(s, \tau) = \frac{1}{5}(a + 3s + 5\tau)$$

Comparing (5.55) with (5.60) by considering (5.62) and (5.63) yields:

$$(5.64) \quad \tau = d - \frac{2}{5}a - \frac{1}{5}s$$

The comparison of (5.56) with (5.61) by taking into account (5.62), (5.63) and (5.64) leads to:

$$(5.65) \quad s = \frac{4}{3}a$$

Substituting (5.65) into (5.64) yields the optimal tax given by:

$$(5.66) \quad \tau = d - \frac{2}{3}a$$

One can rewrite (5.60) and (5.61) as follows:

$$(5.67) \quad p^m - \tau - (2q + r) = 0$$

$$(5.68) \quad p^m + s + \tau - (q + 2r) = 0$$



The total production deriving from (5.67) and (5.68) is given by:

$$(5.69) \quad Q^{m/\tau s} = \frac{1}{3}(2p^m + s)$$

Let us compare  $Q^m$  and  $Q^{m/\tau s}$ . Then, we have:  $\frac{1}{3}(2p^m + s) = 2p^m$  if  $s = 4p^m$ , where  $s = \frac{4}{3}a$  and  $p^m = \frac{1}{3}a$ . One conclude that  $s = 4p^m$ . Hence,

$$(5.70) \quad Q^{m/\tau s} = Q^m$$

The introduction of the tax-subsidy scheme does not modify the total production.

### 5.10. Appendix II: A two-period model

In the preceding lines, we have assumed that the firm extracts and recycles the resource simultaneously over one period. Now, let us relax this assumption and consider that the firm extracts the resource over two periods and recycling occurs in the second-period. The introduction of the sequentiality is more realistic in that recycling derives always from a stock of extracted phosphorus. We also assume that the resource is exhausted over the two periods. Let  $q_1$  denote the quantity of extracted phosphorus in the first-period and  $r$  the quantity of recycled phosphorus in the second-period. Since the resource is exhausted in the second-period, the quantity of extracted phosphorus in this period is  $q_2 = S - q_1$ , where  $S$  is the stock of phosphorus. We assume that only  $q_1$  is recycled. Since full recycling is technically impossible, we have  $r < q_1$ . We assume that recycling does not end up into waters. Then, it reduces or limits pollution in the world of this two-period model. As in the one-period model, the socially efficient solution will set, here, the benchmark.

### 5.10.1. The first-best

Here, the firm is taxed over the two periods, since extracted phosphorus pollutes over both periods. The firm is subsidized only in the second-period, where it recycles. We assume that it incurs a cost of production  $c(q_1)$  in the first-period, a cost of production  $c(S - q_1)$  and a cost of recycling  $c(r)$  in the second-period. We assume that the discount factor is equal to 1. Then, the regulator maximizes the following welfare function which is the sum of consumers surplus and the firm's profits.

$$(5.71) \quad W(q_1, r) = \int_0^{q_1} p(u) du - c(q_1) + \int_0^{S-q_1+r} p(z) dz - c(S - q_1) - c(r) - d[\varepsilon(q_1) + \varepsilon(S - q_1) - \delta(r)],$$

$$(5.72) \quad r < q_1$$

where  $q_1$  is the first-period supply and  $S - q_1 + r$  the second-period supply. If the firm behaves as a price taker, the lagrangian for the programme above yields:

$$(5.73) \quad \mathcal{L}(q_1, r, \lambda) = \int_0^{q_1} p(u) du - c(q_1) + \int_0^{S-q_1+r} p(z) dz - c(S - q_1) - c(r)$$

$$(5.74) \quad -d[\varepsilon(q_1) + \varepsilon(S - q_1) - \delta(r)] + \lambda(q_1 - r),$$

where  $\lambda$  is the lagrange multiplier. The first-order necessary conditions are given by:

$$(5.75) \quad \frac{\partial \mathcal{L}(q_1, r, \lambda)}{\partial q_1} = p_1 - c'(q_1) + c'(S - q_1) - d\varepsilon'(q_1) + d\varepsilon'(S - q_1) + \lambda = 0$$

$$(5.76) \quad \frac{\partial \mathcal{L}(q_1, r, \lambda)}{\partial r} = p_2 - c'(r) + d\delta'(r) - \lambda = 0$$

$$(5.77) \quad \lambda(q_1 - r) = 0,$$

where  $p_1$  and  $p_2$  are the prices charged by the firm respectively in the first and second periods. After the implementation of the tax-subsidy scheme, the profit of the firm is given by:

$$(5.78) \quad \pi(q_1, r) = p_1 q_1 + p_2 \cdot (S - q_1 + r) - c(q_1) - c(S - q_1) - c(r) - \tau[\varepsilon(q_1) + \varepsilon(S - q_1) - \delta(r)] + sr$$

$$(5.79) \quad r < q_1$$

The langrangian for the maximization problem above yields:

$$(5.80) \quad \mathcal{L}(q_1, r, \lambda) = p_1 \cdot q_1 + p_2 \cdot (S - q_1 + r) - c(q_1) - c(S - q_1) - c(r)$$

$$(5.81) \quad -\tau[\varepsilon(q_1) + \varepsilon(S - q_1) - \delta(r)] + sr + \lambda(q_1 - r)$$

The first-order conditions are:

$$(5.82) \quad \frac{\partial \mathcal{L}(q_1, r, \lambda)}{\partial q_1} = p_1 - p_2 - c'(q_1) + c'(S - q_1) - \tau \varepsilon'(q_1) + \tau \varepsilon'(S - q_1) + \lambda = 0$$

$$(5.83) \quad \frac{\partial \mathcal{L}(q_1, r, \lambda)}{\partial r} = p_2 - c'(r) + \tau \delta'(r) + s - \lambda = 0$$

$$(5.84) \quad \lambda(q_1 - r) = 0$$

The comparison of (5.75) with (5.82) and (5.76) with (5.83) induces the tax-subsidy scheme of the form:

$$(5.85) \quad \tau = d$$

$$(5.86) \quad s = 0$$

Since there is one type of distortion, i.e. the negative externality generated here by the first-period and the second-period extractions, the only implementation of the tax suffices to reach the first-best.

Now assume that the firm behaves as a monopolist. Then, the prices depend on the total quantity sold by the firm in each period, and equations (5.75) and (5.76) turn, respectively, into:

$$(5.87) \quad \frac{\partial \mathcal{L}(q_1, r, \lambda)}{\partial q_1} = p_1(q_1) - p_2(S - q_1 + r) - c'(q_1) + c'(S - q_1) - d\varepsilon'(q_1) + d\varepsilon'(S - q_1) + \lambda = 0$$

$$(5.88) \quad \frac{\partial \mathcal{L}(q_1, r, \lambda)}{\partial r} = p_2(S - q_1 + r) - c'(r) + d\delta'(r) - \lambda = 0$$

After the implementation of the tax-subsidy scheme, conditions (5.82) and (5.83) become:

$$(5.89) \quad \frac{\partial \mathcal{L}(q_1, r, \lambda)}{\partial q_1} = p_1'(q_1)q_1 + p_1(q_1) - p_2(S - q_1 + r)$$

$$(5.90) \quad -p_2'(S - q_1 + r) \cdot (S - q_1 + r) - c'(q_1)$$

$$(5.91) \quad -\tau\varepsilon'(q_1) + \tau\varepsilon'(S - q_1) + \lambda = 0$$

$$(5.92) \quad \frac{\partial \mathcal{L}(q_1, r, \lambda)}{\partial r} = p_2(S - q_1 + r) + p_2'(S - q_1 + r) \cdot (S - q_1 + r) - c'(r) + \tau\delta'(r) + s - \lambda = 0$$

The comparison of (5.87) with (5.89) and (5.88) with (5.92) yields the tax-subsidy scheme of the form:

$$(5.93) \quad \tau = d + \frac{p_1'(q_1)q_1 - p_2'(S - q_1 + r) \cdot (S - q_1 + r)}{\varepsilon'(q_1) - \varepsilon'(S - q_1)}$$

$$(5.94) \quad s = \delta'(r) - \frac{p_1'(q_1)q_1}{\varepsilon'(q_1) - \varepsilon'(S - q_1)} - p_2'(S - q_1 + r) \cdot (S - q_1 + r) \left[ 1 - \frac{1}{\varepsilon'(q_1) - \varepsilon'(S - q_1)} \right]$$

Equation (5.93) and (5.94) state that the level of the tax and that of the subsidy with respect, respectively, to the marginal social damage and to the marginal emission of recycling are ambiguous. The tax can be lower or higher than the marginal social damage, according to the level of the distortion deriving from the market power of the monopoly and to that of the marginal emission of extraction. This contradicts the conventional wisdom that the tax applied to a monopolist must be lower than the marginal social damage of pollution.

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### General conclusion

Phosphorus is an ingredient which is vital for life. In addition to being a key component of DNA, and controlling respiration, phosphorus is crucial for agricultural production. Indeed, it plays a major role in many key processes such as photosynthesis, respiration, the storage, the energy transfer and the cell division (Mullins, 2000). It enables a rapid growth of the root system of plants, a good rigidity of the plants and a precocity of the fruits (Bello, 2010). It fosters also resistance of the plants to winter destruction (Mullins and Hajek, 1997). In a nutshell, phosphorus enables to increase agricultural yields and to ensure food security. It is also noteworthy to stress that, based on current scientific knowledge, phosphorus has no substitute in agriculture.

With nitrogen, they are today, the most used fertilizers. But, in contrast to nitrogen which is abundant in the atmosphere and which can be fixed by plants (Weikard and Seyhan, 2009), phosphorus that comes from phosphate reserves is in the control of only a handful of countries, including Morocco, China, Jordan, United States of America, Algeria, Russia, Israel and Senegal. Due to the increasing needs of United States of America in phosphorus and an increasing domestic demand of China, Morocco is the top exporter of phosphorus in the world and exports 35 – 40% of world exports.

Due to population growth which has generated a rise of phosphate fertilizers's demand, phosphorus might be exhausted in the future. Cordell and al. (2009) argue that the world phosphate reserves will be exhausted in another 50 – 100 years. Vaccari (2009) estimates that phosphate reserves will run out in 90 years. Steen (1998) highlights that economically-exploitable reserves will be depleted in 60 – 130 years, whereas Van Kauwenbergh (2010), who is more optimistic, stresses that world phosphate reserves will be exhausted in 300 – 400 years. It is noteworthy to mention that, before the exhaustion of phosphorus, experts predict that a peak of phosphorus will occur in 2033 (Craswell

and al., 2010). This peak corresponds to the point in time at which the global demand of phosphorus will exceed its global production. From that moment on, the quality of phosphorus will decline, its price will increase continuously, it will be very costly to extract and the farmers will have difficult access to this resource. The combination of the crucial role played by phosphorus and the prospect of rarefaction of phosphorus bring up questions about the development of alternatives to its depletion. Accordingly, many solutions, consisting of increasing the price of phosphorus, investing in research and development in order to discover new reserves, improving the efficiency in the extraction process by improving the technology and recycling of phosphorus, have been identified. In the present thesis, we have focused on the latter solution, i.e. recycling of phosphorus. It consists of converting phosphorus contained in wastewater, ashes of sewage sludge, human and animal excreta. Recycling of phosphorus may contribute to the postponement of the depletion of the resource and may reduce water pollution.

In addition to rethink the market of phosphorus in an imperfectly framework, the main aim of this thesis has been to investigate the effect of recycling on the path of extraction of an exhaustible resource holder, on the dynamic of the price of the resource and on the reduction of water pollution usually caused by extracted phosphorus. In order to achieve this goal, we have organized this thesis work around five chapters. The first chapter has stressed the crucial role of the level of the stock in the relationship which exists between extraction and recycling. Indeed, using a model "à la Stackelberg" extended over two periods, it has shown that if the level of the stock is sufficiently small, the monopolist extracts the whole resource in the first-period and the extracted quantity does not depend on recycling. By contrast, if the level of the stock is intermediate, phosphorus is depleted over the two periods and the monopolist's optimal extracted

quantities depend on the existence of recycling. In this situation, its second-period extraction decreases in recycling, whereas its first-period extraction increases in recycling. If the stock is sufficiently large, phosphorus is not exhausted over the two periods and the extracted quantities depend on the recycled quantity. Consequently, the monopolist's extracted quantities decrease with recycling.

In order to observe the continuous dynamic of the extraction or of the price of the resource, we have extended the first chapter in a continuous dynamic framework. We have analyzed, in the second chapter, the effect of recycling of phosphorus on the extraction of phosphate reserves of a monopolist, on the date of exhaustion of phosphorus, on the dynamic of the price of the resource and on consumers'surplus. We have postulated an optimal control model and have found the following results. First, the price increases through time if the level of recyclability is low. Second, the price decreases then increases if the level of recyclability is high. This result is at odds with the conclusion of Hotelling (1931) that the price of exhaustible resources increases over time. Third, the higher the recyclability rate, the more extraction and the exhaustion date are delayed. Fourth, a higher recyclability rate leads to an increase in price in the short-run (a decrease of consumers'surplus in the short run) while it decreases after. It is noteworthy to mention that, in the two previous chapters, we have assumed that extracted and recycled products are strategic substitutes.

The third chapter has relaxed this assumption by considering that extraction and recycling can be either strategic substitutes or strategic complements. It has indicated that the possibility of recycling affects the second-period marginal revenue of the monopolist and the quantities it extracts, and the effect of recycling depends on the strategies exhibited by both products. Indeed, if the quantities are strategic complements, recycling increases the second-period marginal revenue of the monopolist. This increase

triggers the rise of its second-period extraction, resulting in the decline of its first-period extraction. Conversely, if the quantities are strategic substitutes, the effect of recycling is ambiguous. In this case, it depends on whether the strategic effect dominates the recycling capacity effect, and vice versa. If the latter outweighs the former, recycling increases the second-period marginal revenue of the monopolist and reduces its first-period extraction. Conversely, if the former dominates the latter, recycling decreases the second-period marginal revenue of the monopolist and increases its first-period extraction. It should be noted that, in the previous chapters, we have assumed that the owner of the resource always accommodates recycling.

The fourth chapter has shown that the incumbent firm can have an incentive to accommodate recycling as it can have the incentive to prevent the entry of the potential entrant. Also, depending on whether the fixed costs that it incurs are high or low, the potential entrant may decide to enter or to stay out. In contrast to the previous chapters, we have assumed that the incumbent can behave as a competitive firm or as a monopolist. We have used a two-period model, and have considered that the resource is exhausted over the two periods. The two agents have competed "à la Stackelberg". In this chapter, we have, first, analyzed on what conditions, the incumbent will accommodate recycling. Second, we have explored the effect of recycling on the first-period extraction of the monopolist, if it accommodates recycling. Third, we have investigated whether Hotelling's rule may be amended in the presence of recycling or not. As a result, we have shown that when the incumbent behaves as a competitive firm, two scenarios arise. If the fixed costs incurred by the recycler are low, the incumbent accommodates recycling by increasing its first-period extraction. Conversely, if the fixed costs are high, the incumbent must reduce its first-period extraction in order to foster the entry of the recycler. When the incumbent behaves as a monopolist, two cases arise also. If the fixed

costs are low, the monopolist can either ignore recycling by thinking that the latter is irrelevant, or deter it. Indeed, it ignores recycling when the drop in the future price of the resource suffices to discourage recycling. To deter recycling, the monopolist can increase its first-period extraction in order to reduce the future price of the resource; such behavior deters recycling. We have shown that entry deterrence is the best strategy for the extractor. Conversely, if the fixed costs are low, the resource is so scarce that recycling cannot be avoided, the monopolist accommodates recycling and reduces its first-period extraction in order to soften the future competition via the reduction of recycling. We have also shown that the Hotelling's rule must always be amended in the presence of recycling.

In contrast to the four previous chapters, the fifth chapter has taken into account both the polluting nature of extracted phosphorus and the green nature of recycled phosphorus. This chapter has analyzed the role of an environmental tax-subsidy scheme as an instrument for preserving phosphate reserves and for improving water quality by reducing eutrophication. Toward these goals, we have used a model where one firm that can behave as a price taker or as a price maker extracts and recycles phosphorus, simultaneously. We have assumed the presence of a benevolent government that regulates the market by taxing extracted phosphorus and subsidizing recycled phosphorus, simultaneously. First, we have shown that taxing extracted phosphorus and subsidizing recycled phosphorus contribute to the postponement of the depletion of the resource and to the reduction of pollution. Second, we have shown that, in the case where the firm behaves as a price taker, only a Pigovian tax is necessary and it enables to achieve the first-best. Conversely, if the firm is a price-maker, the combination of the two policies is needed. Third, we have shown that the tax-subsidy scheme does not modify the overall

production supplied by the producer. Fourth, we have shown that the structure of the market is determinant in the ways to set the rate of the subsidy.

The contributions of this thesis can be summed up as follows. First, in contrast to the earlier literature, we have shown that the effect of recycling on the pace of extraction of phosphorus is sensitive to the size of the reserves held by the monopolist. Second, this thesis has called into question the conclusion of Hotelling that the price of exhaustible resources increases over time. We have shown that this conclusion does not hold when recyclability is high. Instead of following an upward phase, the price of the resources can decrease in the recycling rate. Third, we have considered the possibility that extracted and recycled products exhibit strategic complementarity. This consideration is important in the sense that it reverses the earlier results and enables to revisit the concept of green paradox which has received special attention in the academic literature. Indeed, in this case, the second-period marginal revenue of the monopolist increases in recycling. Anticipating this increase, the monopolist increases its second-period production. This reduces, mechanically, its first-period production. Such a result is at odds with the result established within the context of the green paradox which states that the eventual presence of a future substitute tends to lead the monopolist or the incumbent firm to increase its current production. Fourth, this thesis is the first one, to the best of our knowledge, to consider that the extraction sector facing a competitive fringe of recyclers can have a structure which is not monopolistic. In the relationship between extraction and recycling, it is the first to have considered that the extractor does not always accommodate recycling. Indeed, one can easily imagine that the holders of the natural resource will use strategies which will prevent the recycler's entry. Fifth, in chapter five, we have considered the polluting nature of phosphorus, and we have studied policies that help to reduce water pollution.



There are some important extensions that we need to explore:

(i) Several chapters will include non null costs of extraction and recycling.

(ii) We have assumed, except the second chapter, one period or two-period settings.

In the future, we will consider multi-period or continuous time versions of the models.

(iii) We have not taken into account the fact that the quality of phosphate reserves is declining over time. It would be useful to postulate a vertical differentiation model which would take into account the low quality and the high quality of phosphorus.

## Conclusion générale

Le phosphore est un ingrédient indispensable à la vie. En plus d'être un élément formant la structure de l'ADN, pilotant la respiration, le phosphore est indispensable à la production agricole. En effet, il joue un rôle majeur dans plusieurs processus clés tels que la photosynthèse, la respiration, le stockage et le transfert de l'énergie (Mullins, 2000). Il favorise également la croissance des racines, la maturité des plantes au début, la résistance aux maladies de pourriture des racines (Mullins et Hajek, 1997). En somme, il permet d'accroître les rendements agricoles, et de contribuer à la sécurité alimentaire. Il convient de souligner qu'en l'état actuel des choses, le phosphore n'a pas de substitut pour son utilisation dans l'agriculture.

Le phosphore et l'azote sont, à l'heure actuelle, les engrais auxquels les agriculteurs ont le plus recours. Mais contrairement à l'azote qui est abondant dans l'atmosphère et qui peut être fixé par quelques plantes (Weikard et Seyhan, 2009), le phosphore provient, pour une grande partie, des roches phosphatées qui ne sont détenues que par une minorité de pays au rang desquels figurent le Maroc, la Chine, la Jordanie, les Etats-Unis, l'Algérie, la Russie, l'Israël et le Sénégal. En raison des besoins américains énormes en phosphore et d'une demande domestique chinoise très forte, le Maroc est le plus grand exportateur mondial et alimente le marché mondial en phosphore à hauteur de 35 – 40% des exportations mondiales.

Du fait de la démographie mondiale galopante qui a engendré une hausse de la demande des engrais phosphatés, le phosphore pourrait s'épuiser dans le futur. Certaines études ont prévu son extinction dans les 50 – 100 ans (Cordell et al., 2009). Un rapport de Vaccari (2009) souligne que les réserves de phosphate s'épuiseront dans 90 ans. D'autres l'ont prévue dans les 60 – 130 ans (Steen, 1998). Van Kauwenbergh (2010), plus optimiste, révèle que les réserves de phosphate s'épuiseront dans 300 – 400 ans.

Faudrait-il noter qu'avant son épuisement, il est prévu que la demande excèdera la production du phosphore en 2033 (Craswell et al., 2011). Ce point correspond à ce qui est appelé, dans la littérature, « le pic du phosphore ». A partir de ce moment, le phosphore va perdre en qualité, son prix va continuellement augmenter, il deviendra plus coûteux de l'extraire et les agriculteurs y accéderont difficilement.

Le rôle crucial joué par le phosphore et la perspective d'une raréfaction de la ressource amènent aujourd'hui à s'interroger sur l'élaboration des alternatives à son épuisement. C'est à cet effet que plusieurs solutions, allant de l'augmentation du prix du phosphore à l'investissement dans la recherche-développement en vue d'explorer de nouvelles réserves et de l'amélioration de l'efficacité dans l'extraction en améliorant la technologie, en passant par le recyclage, ont été proposées dans la littérature. Dans cette thèse, nous nous sommes focalisés sur la solution qui a consisté à recycler le phosphore. Le recyclage consiste à récupérer le phosphore à partir des eaux usées, à partir des déchets humains, à partir des boues d'épuration, etc. L'usage du phosphore recyclé permettrait de différer l'extraction des roches de phosphate et de diminuer la pollution aquatique.

En plus de vouloir reconsidérer le marché du phosphore dans un cadre de concurrence imparfaite, l'objectif visé dans cette thèse a été principalement d'analyser l'effet du recyclage sur l'extraction d'un détenteur de la ressource, sur la dynamique du prix de la ressource et sur la réduction de la pollution de l'eau habituellement causée par le phosphore extrait. Pour atteindre cet objectif, nous avons organisé ce travail de thèse autour de cinq chapitres:

Le premier chapitre a souligné l'importance que la taille des réserves de phosphore peut jouer dans la relation qui existe entre l'extraction et le recyclage. En effet, en utilisant un modèle de concurrence à la Stackelberg étendu sur deux périodes, il a montré que si les réserves détenues par le monopole sont très petites, ce dernier extrait la totalité

de la ressource à la première période et le recyclage n'influence pas l'extraction. En revanche, si les réserves sont intermédiaires, le monopole extrait toute la ressource sur les deux périodes. Dans ce cas, le recyclage a un effet négatif sur l'extraction de deuxième période du monopole compte tenu du fait que le phosphore extrait et le phosphore recyclé sont des substituts stratégiques. La baisse de l'extraction de deuxième période entraîne, mécaniquement, l'augmentation de l'extraction de première période. Si la taille des réserves est assez élevée, la ressource n'est pas épuisée sur les deux périodes. Dans cette situation, le recyclage a toujours un impact négatif sur l'extraction de deuxième période tandis que l'extraction de première période diminue, ici, sous l'effet du recyclage.

Pour pouvoir observer la dynamique continue de l'extraction ou du prix de la ressource, nous avons repensé, dans le chapitre deux, le premier modèle dans un cadre de dynamique continue. Nous avons analysé, dans le deuxième chapitre, l'effet du recyclage du phosphore sur l'extraction du monopole, sur la date d'épuisement du phosphore, sur la dynamique du prix de la ressource et sur le surplus des consommateurs. Nous avons eu recours à un modèle de contrôle optimal et avons montré les résultats suivants. Premièrement, si le taux de recyclage est bas, le prix de la ressource augmente au fil du temps. Deuxièmement, si le taux de recyclage est, en revanche, élevé, le prix de la ressource diminue dans le court terme avant d'augmenter dans le long terme. Ce résultat est en porte-à-faux avec celui établi par Hotelling (1931) pour qui le prix d'une ressource naturelle épuisable augmente continuellement avec le temps. Troisièmement, nous avons montré dans ce chapitre, que plus le taux de recyclage est élevé, plus l'extraction du phosphore est différée dans le temps et plus la date d'épuisement de la ressource est prolongée. Quatrièmement, une augmentation du taux de recyclage conduit à une baisse du surplus des consommateurs dans le court terme, en raison de l'augmentation du prix de la ressource et à une augmentation de ce surplus dans le long terme, due à une baisse

du prix de la ressource. Il est important de noter que, dans les deux premiers chapitres, nous avons fait l'hypothèse que le phosphore extrait et le phosphore recyclé sont des substituts stratégiques.

Dans le troisième chapitre, nous avons considéré que l'effet du recyclage sur la recette marginale de deuxième période du monopole ou sur les quantités qu'il extrait, dépend de la nature des stratégies des deux produits. En effet, si les quantités sont complémentaires, le recyclage augmente la recette marginale de deuxième période du monopole. Cette augmentation provoque celle de son extraction de deuxième période qui, à son tour, entraîne la baisse de son extraction de première période. En revanche, si les quantités sont substituables, l'effet du recyclage est ambigu. Dans ce cas de figure, il dépend de l'effet le plus fort entre l'effet stratégique et le "recycling capacity effect" que l'on peut traduire par l'effet du recyclage. Si le dernier est plus fort, la recette marginale de deuxième période du monopole augmente et la quantité de première période diminue. A l'inverse, si l'effet stratégique l'emporte sur l'effet du recyclage, la recette marginale de deuxième période diminue sous l'effet du recyclage. Une telle baisse provoque la hausse de l'extraction de première période. Il convient de noter que, dans les trois premiers articles, nous avons supposé que le détenteur de la ressource accepte toujours l'entrée du recyclage et s'adapte en conséquence.

Le quatrième chapitre a montré que le détenteur de la ressource peut avoir une incitation à s'adapter à l'entrée du recycleur comme il peut vouloir empêcher son entrée. Aussi, en fonction de la taille des coûts fixes supportés par le recycleur, il peut naturellement décider par lui-même de ne pas rentrer sur le marché. A la différence des chapitres précédents, nous avons considéré ici que l'extracteur peut se comporter comme une entreprise concurrentielle ou comme une entreprise monopolistique. Nous avons utilisé un modèle à deux périodes, avons considéré que la ressource est épuisée à la deuxième

période. Les deux agents ont adopté un mode de concurrence à la Stackelberg. Dans ce chapitre, nous avons cherché, premièrement, à voir sous quelles conditions le détenteur de la ressource accepte l'entrée du recycleur sur le marché. Deuxièmement, il s'est agi d'explorer l'effet du recyclage sur l'extraction de première période du détenteur de la ressource dans le cas où il s'adapte à l'entrée du recycleur. Troisièmement, il s'est agi de voir si la règle d'Hotelling est perturbée par la présence du recyclage ou non. Nous avons montré que, lorsque le secteur d'extraction se comporte comme une entreprise concurrentielle, deux scénarii se présentent. Si les coûts fixes supportés par le recycleur sont faibles, le secteur d'extraction s'adapte à l'entrée en augmentant la quantité qu'il extrait à la première période. En revanche, si les coûts fixes que le recycleur supporte sont élevés, le secteur d'extraction doit réduire son extraction de première période pour encourager l'entrée de recycleur. Dans le cas où le secteur d'extraction se comporte comme un monopoleur, deux situations se présentent également. Si les coûts fixes supportés par le recycleur sont faibles, le monopoleur peut soit ignorer le recyclage en se comportant comme si ce dernier n'est pas rentable, soit le dissuader. En effet, il ignore le recyclage lorsque la baisse du prix futur de la ressource est suffisante pour décourager le recycleur à entrer sur le marché. Pour dissuader l'entrée, le monopoleur peut augmenter son extraction de première période en vue de faire baisser le prix futur de la ressource, ce qui n'incite pas le recycleur à entrer sur le marché. Nous avons montré que la dissuasion est la meilleure stratégie pour le secteur d'extraction. En revanche, si en plus des coûts fixes faibles, la ressource est tellement rare que le recyclage ne peut être évité, le monopoleur s'adapte au recyclage et réduit son extraction de première période dans le but d'atténuer la concurrence future via la réduction du recyclage. Aussi, avons-nous montré que la règle d'Hotelling doit toujours être amendée en présence du recyclage.

Le cinquième chapitre, quant à lui, a pris en compte l'aspect polluant du phosphore extrait et a considéré que le recyclage peut atténuer la pollution engendrée ou peut même faire disparaître la pollution. Il a cherché à analyser le rôle de la combinaison d'une taxe et d'une subvention dans la conservation des réserves de phosphate et dans l'amélioration de la qualité de l'eau, via la réduction de l'eutrophisation. Pour atteindre ces objectifs, nous avons utilisé un modèle où une firme qui peut se comporter soit comme une entreprise concurrentielle soit comme une entreprise monopolistique extrait et recycle à la fois le phosphore. Nous supposons la présence d'un gouvernement bienveillant qui régule le marché en taxant le phosphore extrait et en subventionnant le phosphore recyclé. Premièrement, nous avons trouvé que la combinaison de ces deux politiques contribue à prolonger la durée de vie du phosphore et à réduire la pollution aquatique. Deuxièmement, nous avons montré que, si la firme se comporte comme une entreprise concurrentielle, seule une taxe pigouvienne est nécessaire et elle permet d'atteindre la solution de premier rang. En revanche, si la firme se comporte comme une entreprise monopolistique, il faut combiner les deux instruments, à savoir la taxe et la subvention. Dans ce cas, la taxe est moins élevée que le dommage marginal. Troisièmement, nous avons prouvé que la combinaison des deux instruments ne modifie pas la production totale offerte par la firme. Quatrièmement, nous indiquons que la structure du marché est déterminante dans la manière de fixer le niveau de la subvention.

L'approche originale a, premièrement, consisté à reconsidérer le marché mondial du phosphore dans un cadre de concurrence imparfaite. Cette considération n'est pas anodine dans la mesure où elle fait émerger des interactions stratégiques. Deuxièmement, cette thèse permet d'expliquer pourquoi la prévision faite par Hotelling sur le fonctionnement des marchés des ressources naturelles épuisables n'est pas vérifiée. En effet, Hotelling stipule que plus la ressource naturelle tend vers l'épuisement, plus le prix auquel

elle est vendue croît indéfiniment. Ici, nous avons montré que le recyclage fait que le prix de la ressource peut suivre une phase descendante. Troisièmement, cette thèse est, à notre connaissance, la première, si ce n'est la seule, à avoir considéré que le phosphore extrait et le phosphore recyclé peuvent être des substituts stratégiques ou des compléments stratégiques. Ces considérations ont permis de revisiter le cas Alcoa qui a défrayé la chronique aux Etats-Unis au lendemain de la deuxième guerre mondiale, et le concept de "green paradox" qui a reçu une attention particulière dans la littérature académique. Le quatrième apport de cette thèse est qu'elle est la première à considérer que le secteur d'extraction du phosphore peut se comporter comme une frange concurrentielle ou comme une entreprise monopolistique. Aussi, contrairement aux études précédentes, cette thèse est la première à considérer que le secteur d'extraction ne s'adapte pas toujours à l'entrée du secteur de recyclage. En effet, il peut avoir une incitation à mettre en place des stratégies qui empêchent l'entrée du recycleur. Cinquièmement, cette thèse est l'une des rares à avoir théoriquement pris en compte l'aspect polluant du phosphore dans la relation entre le phosphore extrait et le phosphore recyclé. Cette considération a permis de proposer des solutions à l'eutrophisation.

Il y a d'importantes extensions que nous comptons explorer:

(i) Plusieurs chapitres prendront en compte les coûts d'extraction et de recyclage.

(ii) A l'exception du deuxième chapitre, nous avons considéré des modèles à une ou deux périodes. Dans le futur, nous considérerons des modèles à plusieurs périodes ou des modèles à temps continu.

(iii) Nous n'avons pas considéré le fait que la qualité des réserves de phosphate diminue au fil du temps. Il serait intéressant de mettre en place un modèle de différenciation verticale qui prendrait en compte la qualité élevée et la qualité basse du phosphore.



## Abstract

The theoretical literature that deals with phosphorus considers the market of the resource as being perfectly competitive, whereas the reality of this market suggests otherwise. Indeed, several interactions occur in this market. The main aim of this thesis is to rethink this market in an imperfectly framework. More specifically, we analyze the effect of recycling on the extraction of an exhaustible resource, on the dynamic of the resource price, on its date of depletion and on the reduction of water pollution. This thesis consists in a general introduction and five theoretical chapters all dealing with the economics of phosphorus or of exhaustible resources. Chapter 1 considers a two-period model where an extractor and a recycler compete with quantities. We assume that extracted and recycled phosphorus are strategic substitutes. We show that the effect of recycling on the extracted quantities strongly depends on the level of the stock of phosphorus. Chapter 2 extends the previous chapter in a continuous time framework over an infinite horizon. It investigates the effect of phosphorus recycling on the monopolist's extraction and on the dynamic of its price. We postulate an optimal control model and show that the price of the resource does not necessarily increase through time. Chapter 3 considers that extraction and recycling can be either strategic substitutes or strategic complements. In a two-period model, we show that the effect of recycling on the monopolist's second-period marginal revenue and on its extracted quantities depends on whether extracted and recycled products are strategic substitutes or strategic complements. Chapter 4 considers that the extracting sector chooses between accommodating or preventing the recycler's entry. The entry prevention can take two forms: either deterring or blockading. In a two-period model, we show that the strategy of the extractor depends on the level of the fixed costs incurred by the recycler and on whether the resource is scarce or not. Chapter 5 addresses the problems of phosphorus exhaustion and water pollution. We consider one firm that extracts and recycles phosphorus. We investigate the influence of a tax-subsidy scheme. We show that a combination of these two instruments enables to reduce water pollution and to prolong the lifetime of phosphorus.

**Keywords:** Strategic Interactions, Recycling, Phosphorus.

## Résumé

La littérature théorique portant sur le phosphore considère que le marché de la ressource est parfaitement concurrentiel, alors que son fonctionnement montre, en réalité, qu'il en est autrement. En effet, plusieurs interactions stratégiques existent sur ce marché. L'objectif principal de cette thèse est de reconsidérer ce marché dans un cadre de concurrence imparfaite. Il s'agit, particulièrement, d'analyser l'effet du recyclage sur l'extraction d'une ressource épuisable, sur la dynamique du prix de la ressource, sur sa date d'épuisement et sur la réduction de la pollution aquatique. Cette thèse est organisée autour d'une introduction générale et de cinq chapitres théoriques qui s'intéressent tous à l'économie du phosphore ou des ressources épuisables. Le premier considère un modèle à deux périodes où un pays extracteur et un pays recycleur se concurrencent en quantités. Nous supposons que le phosphore extrait et le phosphore recyclé sont des substituts stratégiques. Nous montrons que l'effet du recyclage sur les quantités extraites par le monopole est très sensible au niveau des réserves qui sont détenues par ce dernier. Le deuxième chapitre est une extension en temps continu du premier à horizon infini. Il analyse l'effet du recyclage du phosphore sur l'extraction du monopole et sur la dynamique du prix de la ressource. Nous utilisons un modèle de contrôle optimal et montrons que le prix de la ressource n'augmente toujours pas au fil du temps. Le troisième chapitre considère que l'extraction et le recyclage peuvent être soit des substituts stratégiques, soit des compléments stratégiques. Il considère un modèle à deux périodes et montre que l'effet du recyclage sur la recette marginale de deuxième période du monopole et sur ses quantités extraites dépend de si les quantités extraites et recyclées sont des substituts ou des compléments stratégiques. Le quatrième chapitre montre que le détenteur de la ressource arbitre entre accepter l'entrée du secteur de recyclage et l'empêcher. La dernière stratégie prend deux formes: soit l'extracteur dissuade l'entrée, soit il la bloque. Nous utilisons un modèle à deux périodes et montrons que la stratégie adoptée par le détenteur de la ressource dépend de la taille des coûts fixes du recycleur et du niveau de rareté de la ressource. Le cinquième chapitre s'intéresse aux problèmes d'épuisement du phosphore et de la pollution aquatique. Nous considérons une firme qui extrait et recycle le phosphore. Nous analysons le rôle de la combinaison d'une taxe et d'une subvention. Nous montrons que la combinaison de ces deux instruments permet de réduire la pollution et de prolonger la durée de vie du phosphore.

**Mots-clés:** Interactions Stratégiques, Recyclage, Phosphore.