Chapitre 4 :

Allocation des terres entre une forêt à plusieurs classes d'âge et un usage agricole en présence d'un risque de perturbation et de préférences récursives

L'allocation des terres entre une forêt à plusieurs classes d'âge et un usage agricole est un problème à la fois dynamique et stochastique dans lequel les deux secteurs n'ont pas le même rapport aux dimensions de risque et de temps. La sylviculture est en effet une activité nécessitant une planification sur plusieurs décennies (voire plusieurs siècles pour certaines essences) alors que l'agriculture est principalement basée sur des cycles de production annuels. En outre, les deux secteurs ne sont pas sensibles aux risques de la même manière. Les cultures sont par exemple moins exposées et moins vulnérables que les forêts au risque de tempête (voir Schelhaas et al. (2010)). Ainsi, l'allocation des terres entre forêt et agriculture est un arbitrage qui prend en compte ces dimensions de risque et de temps et qui subit l'influence des préférences associées.

L'objectif du travail présenté dans ce chapitre est la caractérisation des allocations des terres stationnaires entre une activité agricole annuelle sans risque et une forêt à plusieurs classes d'âge, soumise à un risque de perturbation quand le propriétaire terrien a des préférences récursives. En parallèle, cela pose la question de l'interaction entre allocation des terres et gestion forestière à l'état stationnaire. Par ailleurs, ce travail a également pour objectif de déterminer l'influence respective de l'aversion au risque et des préférences intertemporelles sur l'état stationnaire.

La question de l'allocation des terres entre une forêt à plusieurs classes d'âge et un usage alternatif non dynamique a été abordée dans un contexte déterministe par Salo and Tahvonen (2004). Dans leur modèle, les équilibres mixtes sont rendus possibles par l'introduction de rendements marginaux décroissants. En effet, dans le cas où les rendements sont des fonctions linéaires des surfaces, les allocations mixtes n'existent pas hors cas limites (voir Bell et al. (2006)).

A l'inverse, le problème de l'allocation des terres dans un contexte stochastique a été traité par certains modèles ne prenant pas explicitement en compte la gestion d'une forêt à plusieurs classes d'âge. Ainsi, Parks (1995) présente un modèle dynamique d'allocation des terres entre forêt et agriculture dans le cas où les rendements tirés des deux secteurs sont stochastiques, et montre que des allocations mixtes peuvent être stationnaires à condition que les risques respectifs portant sur les deux secteurs ne soient pas positivement corrélés. Cependant, ce modèle ne dit rien de la gestion forestière à l'état stationnaire ni du lien entre allocation des terres et gestion forestière.

Le travail présenté dans ce chapitre se propose de combiner ces deux dernières approches en traitant conjointement les problèmes de l'allocation des terres et de la gestion forestière en présence de risque. L'approche analytique utilisée par Salo and Tahvonen (2004) est transposée dans un contexte stochastique afin d'identifier les déterminants de l'allocation des sols et de la gestion forestière à l'état stationnaire. En parallèle, un modèle numérique de programmation dynamique stochastique est utilisé pour identifier les différents types d'états stationnaires existants et mesurer leur sensibilité à l'aversion au risque et aux préférences intertemporelles du propriétaire terrien.

Les résultats montrent que l'allocation des terres et la gestion forestière à l'état stationnaire dépendent toutes deux du risque et des préférences, et qu'elles sont interdépendantes. Il est également montré qu'allocation des terres et gestion forestière peuvent être alternativement utilisées par le propriétaire terrien pour adopter des comportements de précaution. Cependant, dans le contexte du modèle numérique utilisé dans cette étude, ces comportements de précaution s'expriment uniquement à travers la diversification de l'usage des terres.

Enfin, les résultats montrent que l'aversion au risque et les préférences intertemporelles ont des influences différentes sur l'allocation des terres et qu'il existe des effets d'interaction entre ces deux types de préférences. Ces deux caractéristiques rendent indispensables l'usage des préférences récursives pour étudier le problème de l'allocation des terres dans un contexte dynamique et stochastique.

Land Allocation between a Multiple-Stand Forest and Agriculture under Perturbation Risk and Recursive Preferences

G. Dumollard

Résumé

This study aims to characterize optimal land allocations between a multiple-stand forest and agriculture, as well as the interdependence between land allocation and forest management, when the forest is subject to a risk of perturbation. The landowner is supposed to have recursive preferences, which permits to distinguish between intertemporal preferences and risk aversion. We show analytically that both land allocation and forest management at stationarity depend on the risk and on both types of preferences and that they are interdependent. Moreover, numerical results from a stochastic dynamic programming model shows that land allocation is used as a means of diversification against risk.

Keywords : Land allocation, Forest management, Recursive preferences, Stochastic Dynamic Programming

JEL : C61, C62, D81, Q24

4.1 Introduction

Land allocation between forestry and agriculture is a dynamic and stochastic issue in which the two sectors do not relate in the same way to the time and risk dimensions. Forestry is indeed an activity that requires planning over several decades (or even centuries for certain species) while agriculture is mostly an activity based on annual cycles. In addition, forestry and agriculture are not equally sensitive to given risks. Cropland is for example less exposed and vulnerable than forests to a risk of storms (see Schelhaas et al. (2010)). Therefore, land allocation is a trade-off that should take into account both time and risk dimensions and corresponding preferences.

The objective of the present study is to characterize stationary land allocations between cropland (or any other annual activity) and a forest with multiple age-classes when the forest is subject to a risk of perturbation and the landowner has recursive preferences. This raises the parallel question of the relationship between land allocation and forest management. In addition, the study aims at determining the respective impact of intertemporal preferences and risk aversion on stationary states.

Focusing on stationary states in presence of a perturbation risk is particularly relevant when the probability of this risk is low as the land system is then more likely to converge and to remain in a stationary state. However, in any case, the stationary state is a horizon to which the producer's decisions tend to lead, and is as such a good indicator on the producer's behavior.

In a dynamic and stochastic context, recursive preferences are more general than expected utility preferences as they take into account preferences on the timing of risk resolution and they permit to distinguish between intertemporal preferences and risk aversion.

As for the preferences on the timing of risk resolution, Spence and Zeckhauser (1972) and Kreps and Porteus (1979) show that if a consumer has expected utility preferences on a dynamic flow of consumption and he can arbitrate between consumption and savings, then his "induced" preferences on the dynamic flow of income cannot be properly described by an expected utility framework. Expected utility preferences are indeed indifferent to the timing of risk resolution, that is to say to the moments when the consumer learns about given random income realizations. In this case, this assumption is unrealistic because prior knowledge of realized incomes enables the consumer to better plan his consumption stream, and should therefore be preferred.

Kreps and Porteus (1978) propose a class of recursive utility functions that are able to represent preferences as to the timing of risk resolution and can then overcome the problems raised by expected utility preferences. Building on this, Epstein and Zin (1989) propose a parameterized sub-class of Kreps and Porteus functions, which allows them to disentangle risk and time preferences, as each of those are represented through distinct parameters. Epstein and Zin (1991) use a function of this sub-class to formulate and estimate a generalized CAPM (Capital Asset Pricing Model). This function is used in the present study.

As for the literature on land allocation, Bell et al. (2006) discuss the main theoretical concepts that underlie land allocation issues. The main point is that land allocation is related to the concept of returns associated to the different land uses. When returns are linear functions of land acreages, the resulting land allocation is a corner solution, which means that all land is allocated to the use presenting the highest return per land unit. On the contrary, diminishing marginal returns (relative to acreages) may result in mixed allocations.

In a deterministic context, Salo and Tahvonen (2004) have developed an analytical model of land allocation and forest management in which both sectors feature diminishing marginal returns. The forest management is flexible and allows for multiple stands. The results show that the stationary rotation age of the forest always follows Faustmann's rule. However, the stationary age-class structure is affected by land allocation. Whenever all land is allocated to forestry, there exists a continuum of stationary periodic forests around the normal forest. The existence of such stationary periodic forests have been extensively discussed in Salo and Tahvonen (2002a), Salo and Tahvonen (2002b) et Salo and Tahvonen (2003). However, whenever land allocation is mixed, the forest is normal.

The introduction of diminishing marginal returns in Salo and Tahvonen (2004) is *ad hoc* but in reality they can be justified on different bases, for example the heterogeneity in soil and climate conditions. There is a large empirical literature on land allocation and many articles that deal with soil heterogeneity. For example, Stavins and Jaffe (1990) propose an econometric estimation of unobserved soil quality distribution based on observed land allocation. The econometric model developed by Stavins and Jaffe is structured by a dynamic model of land allocation.

The presence of risk is another factor that can explain mixed land allocations. On the basis of a dynamic and stochastic model of land allocation, Parks (1995) shows that when risks held by two different land uses are not positively correlated and the landowner is risk-averse, then land allocation is mixed. This reflects a diversification behavior.

The aim of this article is two-fold : i) providing analytical results, in particular identifying the determinants of stationary land allocation and forest management, and ii) providing clear numerical evidence on the different types of land allocation and forest management at stationarity. In addition, numerical results are aimed to determine the respective role of intertemporal preferences and risk aversion and to determine which of land allocation or forest management is the lever used as a cover against risk by the landowner.

In section 4.3, the analytical approach proposed in Salo and Tahvonen (2004) and based on Karush-Kuhn-Tucker conditions is adapted to a stochastic context and is used to identify the determinants of stationary states. In section 4.4, a stochastic dynamic programming model is used to identify the different types of stationary states and to assess the sensitivity of land allocation to preference parameters.

4.2 A stochastic land allocation and forest management program

In the model considered in this study, land allocation and forest management are the two endogenous decisions. Forest management is even-aged and allows for multiple age-classes. The even-aged management implies that age-classes are spatially separated and there are subsequently no interaction effects between them in terms of biological growth. Moreover, thinning is ruled out, the landowner can only harvest by clear-cutting. However, at a given time, an age-class may be harvested only on a fraction of the total acreage it covers. In addition, a random perturbation may occur at any time. The occurrence of this perturbation is assumed to destroy completely certain age-classes, with no residual value left, and to leave the other age-classes, as well as the entire agricultural land, completely untouched.

These hypotheses ensure that the state of the land system at a given time can be fully described by the land acreages allocated to agriculture and to the different age-classes of the forest, that are respectively noted $x_{agr,t}$ and $x_{a,t}$ with $t \in \mathbb{N}$ the time index and $a \in \mathbb{N}$ the age-class index.

The model is defined in a discrete time setting and the program faced by the landowner is sequential and repeats at every time-step. At time t, the landowner observes the state of his land system, which is defined by the vector of land shares $X_t = (x_{agr,t}, x_{1,t}, x_{2,t}, ..., x_{a,t}, ...)$. On the basis of this observation, he makes decisions concerning timber harvest, forest planting and land allocation to agriculture. These decisions determine the income from timber harvest received at time t, $\Pi_{for,t}$, as well as the state $X_{t+1} = (x_{agr,t+1}, x_{1,t+1}, x_{2,t+1}, ..., x_{a,t+1}, ...)$ that is realized at $t+1$ when the perturbation does not occur between t and $t+1$. The agricultural income $\Pi_{agr,t}$ received at time t is a function of the agricultural land acreage at time t, $x_{agr,t}$, it does not depend on decisions and is certain from a time t perspective. The forest income is also certain from a time t perspective as harvest decisions are made and executed before the perturbation may occur.

At each time step, the perturbation occurs with probability p or does not occur with probability $(1-p)$, it is an independently and identically distributed Bernoulli trial. If it does not occur, decided land shares $X_{t+1} = (x_{agr,t+1}, x_{1,t+1}, x_{2,t+1}, ..., x_{a,t+1}, ...)$ are realized at $t+1$. If it occurs, decided land shares are modified to become $D(X_{t+1}) = (d_{agr}(x_{agr,t+1}), d_1(x_{1,t+1}), d_2(x_{2,t+1}), ..., d_a(x_{a,t+1}), ...)$. Functions $d_1, ..., d_a, ...$ are functions describing the impact of the perturbation on the different age-classes. Knowing that the perturbation completely destroys given age-classes and leaves the others untouched, then for any age-class a, either $d_a(x_{a,t}) = x_{a,t}$ or $d_a(x_{a,t}) = 0$ depending on how the perturbation is specified. As we assume that agricultural activities are not impacted by the perturbation, we have $d_{agr}(x_{agr,t+1}) = x_{agr,t+1}$. Once state $t+1$ is realized, either X_{t+1} or $D(X_{t+1})$, the landowner observes it and the same sequence reproduces.

This sequential program is represented on the timeline given in figure 15 :

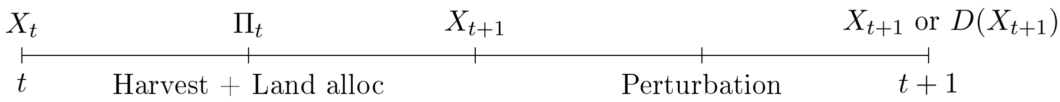

FIGURE 15 – Timeline of decisions and realizations between t and $t+1$

This sequential problem can be analyzed using the framework proposed by Epstein and Zin (1989), which involves similar temporal lotteries. For such temporal lotteries, Epstein and Zin (1989) define a new preference framework based on recursive utility functions. More precisely, Epstein and Zin (1991) define a recursive utility function, U_t, on the basis of the following recursive relation :

$$U_t = \left((1-\beta)\Pi_t^{\frac{\sigma-1}{\sigma}} + \beta[E(\tilde{U}_{t+1}^{1-\alpha})]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}} \right)^{\frac{\sigma}{\sigma-1}} \tag{89}$$

This functional relation combines a Constant Elasticity of Substitution (CES) function and a Constant Relative Risk Aversion (CRRA) expected utility function.

U_t is the utility brought to the landowner by all possible future income streams following time t. This way of aggregating future possible incomes is an alternative to the use of the standard expected utility framework. Recursive preferences actually generalize expected utility preferences, which are obtained as a particular case when $\alpha = \frac{1}{\sigma}$.

Π_t is the certain (from a time t standpoint) income received at time t, $\tilde{U}_{t+1}$ is the uncertain recursive utility brought to the landowner by all possible incomes received from $t+1$ onwards, and E the expectation operator corresponding to our Bernoulli trial. This uncertainty reflects the fact that the sets of possible income streams as of $t+1$ do not need to be the same whether a perturbation occurs between t and $t+1$ or not. $\sigma \in \mathbb{R}^{+*}$ is the intertemporal preferences parameter (the greater σ, the more flexible intertemporal substitutions), $\alpha \in \mathbb{R}^{+}$ is the risk aversion parameter (the greater α, the more risk-averse the producer), and β is the subjective discount rate.

Epstein and Zin (1989) have proven that it is possible to maximize U_t by adjusting, under constraints, all possible future incomes for all possible perturbation scenarios. As the program is sequential, they have also shown that it is possible to determine an optimal decision rule at time t given the observed state of the land system X_t. This decision rule is stationary (independent from t) and denoted $d(X_t)$.

In addition, Epstein and Zin (1989) have demonstrated that this optimal decision rule can be determined using a modified Bellman equation. The general form of this modified Bellman equation, as introduced by Epstein and Zin (1991) is given in equation 90 :

$$V(X_t) = \max_{d \in \mathbb{D}_t} \left\{ \left((1-\beta)\Pi(X_t, d)^{\frac{\sigma-1}{\sigma}} + \beta [E(V((\tilde{X}_{t+1} \mid X_t, d))^{1-\alpha})]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}} \right)^{\frac{\sigma}{\sigma-1}} \right\} \tag{90}$$

Considering state X_t, decisions d must belong to the feasible set $\mathbb{D}_t$. $\Pi(X_t, d)$ is the certain income received by the producer at t from both forestry and agriculture. It depends on both state X_t and decisions d. $(\tilde{X}_{t+1} \mid X_t, d)$ is the distribution over the random state of the land system at $t+1$ conditional on state X_t and decisions d.

The value function $V(X_t)$ that verifies equation 90 gives the maximum value of U_t when the initial state at time t is X_t and when all subsequent decisions are optimal regardless of the perturbation scenario that will be realized.

Equation 90 as well as the feasible decision set $\mathbb{D}_t$ can be further specified in the context of our model.

As for decisions, the acreage of age-class a that is harvested between t and $t+1$ is noted $h_{a,t}$. It corresponds to the difference between the acreage $x_{a,t}$ allocated to age-class a at time t and the acreage $x_{a+1,t+1}$ allocated to age-class $a+1$ at $t+1$ if no perturbation occurs between t and $t+1$, that is the same age-class but one time-step older and one time-step later. The acreage planted with new forest between t and $t+1$ is denoted s_t, it corresponds to the acreage of land allocated to age-class 1 at the next time step if no perturbation occurs by then, that is $x_{1,t+1}$. At last, at time t, the landowner decides to allocate a land share ag_t to agriculture. This allocation is realized with certainty at $t+1$ and corresponds to land share $x_{agr,t+1}$.

Thus, the relation between control variables $h_{a,t}$, s_t and ag_t, and land shares $(x_{agr,t}, x_{1,t}, ..., x_{a,t}, ...)$ observed at t, and land shares $(x_{agr,t+1}, x_{1,t+1}, ..., x_{a,t+1}, ...)$ realized at $t+1$ in the absence of perturbation between t and $t+1$, can be expressed as follows :

$$\text{For all } a \in \mathbb{N}^* \text{ and } t \in \mathbb{N} : \quad h_{a,t} = x_{a,t} - x_{a+1,t+1} \tag{91}$$

$$\text{For all } t \in \mathbb{N} : \quad s_t = x_{1,t+1} \tag{92}$$

$$\text{For all } t \in \mathbb{N} : \quad ag_t = x_{agr,t+1} \tag{93}$$

Therefore, for a given state $X_t = (x_{agr}, x_{1,t}, ..., x_{a,t}, ...)$ observed at t, decisions made between t and $t+1$, that are $(ag_t, s_t, h_{1,t}, ..., h_{a,t}, ...)$, can also be represented by decided land shares $X_{t+1} = (x_{agr,t+1}, x_{1,t+1}, ..., x_{a,t+1}, ...)$ (that will be realized if no perturbation occurs between t and $t+1$) as they contain all the information regarding the decisions.

As mentioned above, the income Π_t is certain from a time t standpoint as both components $\Pi_{agr,t}$ and $\Pi_{for,t}$ are certain.

$\Pi_{for,t}$ can be expressed in function of harvested acreages $h_{a,t}$ or equivalently in function of land shares as follows :

$$\Pi_{for,t} = \sum_{a=1}^{+\infty} R_a h_{a,t} = \sum_{a=1}^{+\infty} R_a (x_{a,t} - x_{a+1,t+1}) \tag{94}$$

R_a is the income generated by harvesting one land unit of age-class a. Note that timber value results from a pure-aging process, which is a consequence of the even-aged management hypothesis. Incidentally, equation 94 reflects the absence of harvest and planting costs in the model.

$\Pi_{agr,t}$ depends only on the agricultural land share at time t, $x_{agr,t}$ and is written as follows :

$$\Pi_{agr,t} = R_{agr} x_{agr,t} \tag{95}$$

R_{agr} is the income generated during one time-step by one agricultural land unit. R_agr is independent from land allocation, which means that we assume constant marginal returns to land. This is a major difference from Salo and Tahvonen (2004) who consider the problem in a deterministic context but with decreasing marginal returns to land.

As a consequence, the total income received by the landowner at time t can be expressed as a function of observed land shares X_t and decided land shares X_{t+1} as shown in equation 96 :

$$\Pi_{agr,t} + \Pi_{for,t} = R_{agr} x_{agr,t} + \sum_{a=1}^{+\infty} R_a (x_{a,t} - x_{a+1,t+1}) = \Pi(X_t, X_{t+1}) \tag{96}$$

In addition, as assumed above, the perturbation is an independently and identically distributed Bernoulli trial which leads to state X_{t+1} with probability $(1-p)$ and to state $D(X_{t+1})$ with probability p.

On the basis of these clarifications, the Bellman equation can be rewritten as follows :

$$V(X_t) = \max_{X_{t+1}} \left\{ \left((1-\beta)\Pi(X_t, X_{t+1})^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)V(X_{t+1})^{1-\alpha} + pV(D(X_{t+1}))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}} \right)^{\frac{\sigma}{\sigma-1}} \right\} \tag{97}$$

Moreover, decided land shares $X_{t+1} = (x_{agr,t+1}, x_{1,t+1}, x_{2,t+1}, ..., x_{a,t+1}, ...)$ are subject to two types of constraints.

The acreages of land $h_{a,t}$ that are harvested from the different age-classes are by definition positive. On the basis of equation 10, these constraints can be expressed as follows :

$$\text{For all } a \in \mathbb{N}^* \text{ and } t \in \mathbb{N} : \quad x_{a,t} \geq x_{a+1,t+1} \tag{98}$$

The total land acreage available for both agricultural and forest production is normalized to one, without loss of generality :

$$\text{For all } t \in \mathbb{N}^* : \quad x_{agr,t} + \sum_{a=1}^{+\infty} x_{a,t} \leq 1 \tag{99}$$

The Bellman equation 97, as well as constraints 98 and 99, define the optimization program faced by the landowner.

This optimization program can be solved numerically using dynamic programming methods as performed in section 4.4. However, it is also possible to derive a few analytical results using Karush-Kuhn-Tucker conditions as shown in section 4.3.

4.3 Analytical characterization of stationary land allocations

Necessary conditions for the stationarity of a mixed land allocation between agriculture and a normal forest is presented in section 4.3.1. This condition is demonstrated in section 4.3.2.

4.3.1 Stationary land allocations and normal forests

We write $X_s = (x_{agr,s}, x_{1,s}, x_{2,s}, ..., x_{a,s}, ...)$ a vector of land shares describing a mixed land allocation between agriculture and a normal forest of rotation age F. Land allocation is mixed when both activities coexist, that is $x_{agr,s} > 0$ and $\sum_{a=1}^{+\infty} x_{a,s} = 1 - x_{agr,s} > 0$. As the forest is normal and has a rotation age F, $x_{a,s} = \frac{1-x_{agr,s}}{F}$ for $a \leq F$ and $x_{a,s} = 0$ for $a > F$.

Π_s is the income generated at each time step by the land system X_s as long as it remains in this state, which means that $\Pi_s = R_{agr}x_{agr,s} + \frac{(1-x_{agr,s})R_F}{F}$ (see equation 96).

V is the value function that verifies the dynamic program defined by the Bellman equation 97 under constraints 98 and 99.

V^e is a function such that $V^e(x_{agr}, x_1, x_2, ..., x_a, ...) = V(d_{agr}(x_{agr}), d_1(x_1), d_2(x_2), ..., d_a(x_a), ...)$ where functions $d_{agr}, d_1, d_2, ..., d_a, ...$ describe the impact of the perturbation on the different land shares (see section 4.2).

Hypothesis 1 *If the land system is initially in a state X_s, then it is optimal that it remains* *in the same state X_s as long as no perturbation occurs.*

Hypothesis 1 states that we consider a mixed land allocation between agriculture and a normal forest of rotation age F, X_s, that is stationary as long as no perturbation occurs.

Hypothesis 2 $\beta(1-p)\left(\frac{[(1-p)V(X_s)^{1-\alpha}+pV^e(X_s)^{1-\alpha}]^{\frac{1}{1-\alpha}}}{V(X_s)}\right)^{\alpha-\frac{1}{\sigma}} < 1.$

Hypothesis 2 allows us to perform the calculations up to conditions 100 and 101 below. It is a weak hypothesis that is for example verified when $\alpha \geq \frac{1}{\sigma}$ (although it is not necessary).

Proposition 1 *Considering a landowner facing the optimization program defined by the Bellman equation 97 and by constraints 98 and 99, then under hypotheses 1 and 2, conditions 100 and 101 necessarily hold :*

For all $j \in \mathbb{N}$:

$$\frac{1}{1-\gamma^F}\left[(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}\gamma^F R_F + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\left(\gamma\frac{\partial V^e}{\partial x_1}\Big|_{X_s} + ... + \gamma^F\frac{\partial V^e}{\partial x_F}\Big|_{X_s}\right)\right]$$
$$\geq \frac{1}{1-\gamma^j}\left[(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}\gamma^j R_j + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\left(\gamma\frac{\partial V^e}{\partial x_1}\Big|_{X_s} + ... + \gamma^j\frac{\partial V^e}{\partial x_j}\Big|_{X_s}\right)\right] \tag{100}$$

$$\frac{\gamma}{1-\gamma}\left[(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}R_{agr} + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\frac{\partial V^e}{\partial x_{agr}}\Big|_{X_s}\right]$$
$$= \frac{1}{1-\gamma^F}\left[(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}\gamma^F R_F + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\left(\gamma\frac{\partial V^e}{\partial x_1}\Big|_{X_s} + ... + \gamma^F\frac{\partial V^e}{\partial x_F}\Big|_{X_s}\right)\right] \tag{101}$$

with :

$$\gamma = \beta(1-p)\left(\frac{[(1-p)V(X_s)^{1-\alpha} + pV^e(X_s)^{1-\alpha}]^{\frac{1}{1-\alpha}}}{V(X_s)}\right)^{\alpha-\frac{1}{\sigma}} \tag{102}$$

Conditions 100 and 101 show that if a mixed land allocation between agriculture and a normal forest X_s is stationary, then both land allocations and forest management depend on the perturbation probability and consequences (parameter p and functions $(d_agr, d_1, ..., d_a, ...)$), on intertemporal preferences (parameter σ) and on risk aversion (parameter α). Indeed, for a given state X_s, when these parameters are changed, conditions 100 and 101 do not necessarily hold anymore and in this case X_s can no longer be stationary.

In addition, both equations 100 and 101 depend on land acreages $X_s = (x_{agr}, x_1, x_2, ..., x_a, ...)$, through the value function V and its derivatives. As a consequence, condition 100 on the rotation age F of the normal forest on land allocation and condition 101 that allows for mixed land allocation depends on the rotation age F. In other words, in a stationary state X_s, forest management and land allocation are interdependent. This observation differs from the results obtained by Salo and Tahvonen (2004) who show in a deterministic setting that forest management always respect Faustmann's rule regardless of land allocation.

Unfortunately, it is not generally possible to derive tractable expressions giving the rotation age F and land allocation in function of exogenous parameters from equations 100 and 101. However, further information can be derived for some particular cases.

When the perturbation is assumed to destroy completely the whole forest and all crops, equations 103 and 104 can be respectively rewritten as follows :

$$F = \arg\max_{j \in \mathbb{N}} \left\{ \frac{\gamma^j}{1-\gamma^j} R_j \right\} \tag{103}$$

$$\frac{\gamma}{1-\gamma} R_{agr} = \frac{\gamma^F}{1-\gamma^F} R_F \tag{104}$$

Indeed, if the perturbation destroys completely the forest and the crops, we have $d_{agr}(x_{agr}) = d_1(x_1) = ... = d_a(x_a) = ... = 0$ for all states $X = (x_{agr}, x_1, ..., x_a, ...)$. Subsequently, $\frac{\partial d_a}{\partial x_a} = 0$ for all a and $\frac{\partial d_{agr}}{\partial x_{agr}} = 0$, then $\frac{\partial V^e}{\partial x_a} = \frac{\partial V}{\partial d_a(x_a)} \frac{\partial d_a(x_a)}{\partial x_a} = 0$ for all a and $\frac{\partial V^e}{\partial x_{agr}} = \frac{\partial V}{\partial d_{agr}(x_{agr})} \frac{\partial d_{agr}(x_{agr})}{\partial x_{agr}} = 0$.

As γ depends on the risk and on preferences, equations 103 and 104 show that the rotation age and land allocation in a stationary state X_s still depend on risk and preferences in presence of a risk of total destruction.

In addition, both equations depend on land shares, which means that equality 104 is not a limit condition on exogenous parameters (see below).

Further simplifying, we assume that the landowner has expected utility preferences, which means that $\alpha = \frac{1}{\sigma}$ (see section 4.2) and therefore that $\gamma = \beta(1-p)$. In this case, equations 103 and 104 reduce to equations 105 and 106 :

$$F = \arg\max_{j \in \mathbb{N}} \left\{ \frac{[\beta(1-p)]^j}{1-[\beta(1-p)]^j} R_j \right\} \tag{105}$$

$$\frac{\beta(1-p)}{1-\beta(1-p)} R_{agr} = \frac{[\beta(1-p)]^F}{1-[\beta(1-p)]^F} R_F \tag{106}$$

Equations 105 and 106 do not depend on land shares but exclusively on exogenous parameters and they are tractable.

Equation 105 shows that the rotation age of the normal forest does not depend on preferences but only on the perturbation probability p. As it is a limit condition on exogenous parameters, it also shows that there is only one possible rotation age for the normal forest (except for limit cases), regardless of land allocation.

Equation 106 is also a limit condition on exogenous parameters, it shows that mixed land allocations cannot exist (except for limit cases) in presence of a risk of total destruction and when the landowner has expected utility preferences while they may exist when the landowner has recursive preferences. This difference between recursive and expected utility preferences is confirmed numerically (see the numerical approach in section 4.4).

This difference between recursive preferences and expected utility preferences in presence of a risk of total destruction is due to the fact that recursive preferences, unlike expected utility preferences, take into account the timing in risk resolution through the term $\left(\frac{[(1-p)V(X_s)^{1-\alpha} + pV^e(X_s)^{1-\alpha}]^{\frac{1}{1-\alpha}}}{V(X_s)} \right)^{\alpha - \frac{1}{\sigma}}$ that appears in γ.

Finally, in the absence of risk, that is when $p = 0$, equations 100 and 101 can be written as follows :

$$F = \arg\max_{j \in \mathbb{N}} \left\{ \frac{\beta^j}{1-\beta^j} R_j \right\} \tag{107}$$

$$\frac{\beta}{1-\beta} R_{agr} = \frac{\beta^F}{1-\beta^F} R_F \tag{108}$$

Equation 107 is Faustmann's rule expressed in a discrete time setting. It is consistent with Salo and Tahvonen (2002b) who show in a deterministic setting that stationary forests (including the normal forest) necessarily follow Faustmann's rule.

Similarly to equation 106, equation 108 is a threshold condition allowing us to distinguish between all-agriculture and all-forestry land allocations. Equation 108 states that in a deterministic setting, all land is allocated to the use offering the highest series of discounted returns. This result is consistent with the corner solutions described by Bell et al. (2006).

4.3.2 Demonstration

In order to prove proposition 1, we use an alternative formulation of the landowner's objective to the Bellman equation.

Recursive relation 109 describes how future possible incomes are aggregated to form the landowner's utility, that is so far not maximized :

$$U_t = \left(((1-\beta)\Pi_t)^{\frac{\sigma-1}{\sigma}} + \beta [E(\tilde{U}_{t+1})^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}} \right)^{\frac{\sigma}{\sigma-1}} \tag{109}$$

U_t is an aggregate of the income Π_t received by the landowner at time t (that is certain from a time t standpoint) and of the utility $\tilde{U}_{t+1}$ brought by all possible incomes received from time $t+1$ onwards. From a time t perspective, the utility $\tilde{U}_{t+1}$ is uncertain. It can either be U_{t+1} if no perturbation occurs between t and $t+1$ (probability $(1-p)$) or U^e_{t+1}

if the perturbation occurs (probability p). U_{t+1} and U^e_{t+1} do not need to be equal as future possible income streams do not need to be the same whether a perturbation has occurred between t and $t+1$ or not.

We consider an initial date $t = 0$, at which the state of the land system is described by the vector of land shares $X_0 = (x_{agr,0}, x_{1,0}, ..., x_{a,0}, ...)$. Then, the landowner makes decisions, that are not necessarily optimal, which leads the system to state $X_1 = (x_{agr,1}, x_{1,1}, ..., x_{a,1}, ...)$ at $t = 1$ if no perturbation occurs between t and $t+1$ or to state $D(X_1) = (d_{agr}(x_{agr,1}), d_1(x_{1,1}), ..., d_a(x_{a,1}, ...))$ if the perturbation occurs. On this basis, recursive relation 109 leads to :

$$U_0 = \left((1-\beta)\Pi(X_0, X_1)^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_1^{1-\alpha} + pU_1^{e\,1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}} \right)^{\frac{\sigma}{\sigma-1}} \tag{110}$$

So far, neither the decisions taken between $t = 0$ and $t = 1$ nor any following decisions are optimized.

However, we now suppose that all decisions taken from $t = 1$ onwards are optimal but only when a perturbation has occurred between $t = 0$ and $t = 1$. After section 4.2, we know that the value function $V(X_t)$ (that verifies the optimization program described by equations 97, 98 and 99) is the maximum recursive utility U_t given an initial state X_t and given the specifications and constraints of our model, that is $V(X_t) = \max\{U_t\},$.

After the perturbation has occurred between $t = 0$ and $t = 1$, the state of the system is $D(X_1)$ and the maximum recursive utility that can be derived starting from this state is $V(D(X_1))$. Consequently, equation 22 can be written as follows :

$$U_0 = \left((1-\beta)\Pi(X_0, X_1)^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_1^{1-\alpha} + pV(D(X_1))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}} \right)^{\frac{\sigma}{\sigma-1}} \tag{111}$$

If the perturbation does not occur, the state of the forest at $t = 1$ is X_1 and the landowner makes decisions to apply between $t = 1$ and $t = 2$ that are not necessarily optimal. Considering these decisions, the state X_2 is realized at $t = 2$ if no perturbation occurs between $t = 1$ and $t = 2$, otherwise state $D(X_2)$ is realized. If we assume that all decisions following $t = 2$ are optimal when the perturbation has not occurred between $t = 0$ and $t = 1$ but that it has occurred between $t = 1$ and $t = 2$, we can write :

$$U_1 = \left((1-\beta)\Pi(X_1, X_2)^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_2^{1-\alpha} + pV(D(X_2))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}} \right)^{\frac{\sigma}{\sigma-1}} \tag{112}$$

The same rationale can be applied again and indefinitely, we can therefore obtain recursive relation 113 for all $t \in \mathbb{N}$:

$$U_t = \left((1-\beta)\Pi(X_t, X_{t+1})^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_{t+1}^{1-\alpha} + pV(D(X_{t+1}))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}} \right)^{\frac{\sigma}{\sigma-1}} \tag{113}$$

Figure 16 shows how this way of expressing the sequential problem can be represented on the binomial tree corresponding to our perturbation :

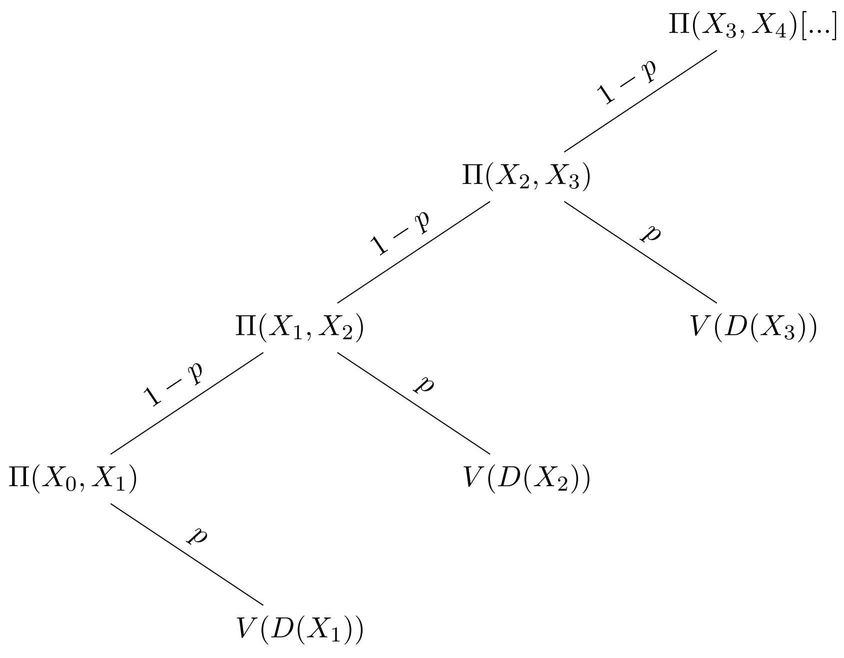

FIGURE 16 – Recursive aggregation and the binomial tree

The fact that U_1 is nested in U_0, U_2 in U_1 and so forth indefinitely provides us with an expression $U_0(X_0, X_1, ..., X_t, ...)$ that depends on the initial state X_0 (constrained) and on vectors of land shares $(X_1, ..., X_t, ...)$ that will be realized if the perturbation never occurs.

$U_0(X_0, X_1, ..., X_t, ...)$ is the objective function of the landowner at time $t = 0$ and $(X_1, ..., X_t, ...)$ are the vector of control variables that can be used to maximize it (under contraints). $(X_1, ..., X_t, ...)$ are the only variables that are left to be maximized as all the decisions following the first occurrence of a perturbation in all other perturbation scenarios are implicitly maximized through the use of the value function V. More generally, a similar objective function $U_t(X_t, X_{t+1}, ...)$ can actually be defined when considering the problem from any initial state X_t starting from any time $t \in \mathbb{N}$.

This approach for expressing U_0 allows us to isolate vectors of land shares $(X_1, ..., X_t, ...)$ that are realized when a perturbation never occurs. This is appropriate as we want to obtain a condition of stationarity for the land system provided that the perturbation never occurs.

The demonstration of necessary conditions 100 and 101 is based on Karush-Kuhn-Tucker conditions. First, the Lagrangian corresponding to the maximization of $U_0(X_0 = X_s, X_1, X_2, ..., X_t, ...)$ (the initial state X_0 is constrained) under constraints 98 and 99 can be written as follows :

$$L(X_1, ..., X_t, ..., \{\mu_{a,t}\}, \{\eta_t\}) = U_0(X_0 = X_s, X_1, ..., X_t, ...) + \sum_{t=0}^{+\infty}\sum_{a=1}^{+\infty} \mu_{a,t}(x_{a,t} - x_{a+1,t+1}) + \sum_{t=0}^{+\infty} \eta_t(1 - x_{agr,t} - \sum_{a=1}^{+\infty} x_{a,t}) \quad (114)$$

$\{\mu_{a,t}\}$ and $\{\eta_t\}$ are the dual variables respectively associated with constraints 98 and 99 introduced in section 4.2.

We assume that the system is initially a mixed land allocation between agriculture and a normal forest of rotation age F, noted $X_s = (x_{agr,s}, x_{1,s}, ..., x_{a,s}, ...)$, that is $X_0 = X_s$. Land allocation is mixed so $0 < x_{agr,s} < 1$ and the forest is normal and has a rotation age F so $x_{a,s} = (1 - x_{agr,s})\frac{1}{F}$ for $a \leq F$ and $x_{a,s} = 0$ for $a > F$.

In addition, we assume that it is optimal that the system remains in this state as long as the perturbation does not occur, which means that decisions $X_1 = X_2 = ... = X_t = ... = X_s$ are optimal when $X_0 = X_s$.

The Lagrangian derivatives evaluated at $(X_0 = X_1 = X_2 = ... = X_t = ... = X_s, \{\mu_{a,t}\}, \{\eta_t\})$ are written as follows :

$$\frac{\partial L}{\partial x_{agr,t}}\Big|_{X_0=X_1=...=X_s,\{\mu_{a,t}\},\{\eta_t\}} = \frac{\partial U_0}{\partial x_{agr,t}}\Big|_{X_0=X_1=...=X_s} - \eta_t \tag{115}$$

For $a > 1$:

$$\frac{\partial L}{\partial x_{a,t}}\Big|_{X_0=X_1=...=X_s,\{\mu_{a,t}\},\{\eta_t\}} = \frac{\partial U_0}{\partial x_{a,t}}\Big|_{X_0=X_1=...=X_s} + \mu_{a,t} - \mu_{a-1,t-1} - \eta_t \tag{116}$$

For $a = 1$:

$$\frac{\partial L}{\partial x_{1,t}}\Big|_{X_0=X_1=...=X_s,\{\mu_{a,t}\},\{\eta_t\}} = \frac{\partial U_0}{\partial x_{1,t}}\Big|_{X_0=X_1=...=X_s} + \mu_{1,t} - \eta_t \tag{117}$$

As demonstrated in appendix 1, recursive relations 113 allow us to determine derivatives $\frac{\partial U_0}{\partial x_{agr,t}}$ and $\frac{\partial U_0}{\partial x_{a,t}}$. In particular, it is demonstrated that the values of $\frac{\partial U_0}{\partial x_{agr,t}}$ and $\frac{\partial U_0}{\partial x_{a,t}}$ when $X_0 = X_1 = ... = X_t = ... = X_s$, can be written as follows :

$$\frac{\partial U_0}{\partial x_{agr,t}}\Big|_{X_0=X_1=...=X_t=...=X_s} = \gamma^t(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_{agr} + \gamma^t \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha} \frac{\partial V^e}{\partial x_{agr,t}}\Big|_{X_s} \tag{118}$$

For $a > 1$:

$$\frac{\partial U_0}{\partial x_{a,t}}\Big|_{X_0=X_1=...=X_t=...=X_s} = \gamma^{t-1}(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(-R_{a-1}) + \gamma^t(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_a + \gamma^t \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha} \frac{\partial V^e}{\partial x_{a,t}}\Big|_{X_s} \tag{119}$$

For $a = 1$:

$$\frac{\partial U_0}{\partial x_{1,t}}\Big|_{X_0=X_1=...=X_s} = \gamma^t(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_1 + \gamma^t \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha} \frac{\partial V^e}{\partial x_{1,t}}\Big|_{X_s} \tag{120}$$

With

$$\gamma = \beta(1-p)\left(\frac{[(1-p)V(X_s)^{1-\alpha} + pV^e(X_s)^{1-\alpha}]^{\frac{1}{1-\alpha}}}{V(X_s)}\right)^{\alpha-\frac{1}{\sigma}} \tag{121}$$

$\Pi_s = \Pi(X_s, X_s) = R_{agr}x_{agr_s} + \frac{(1-x_{agr,s})R_F}{F}$ is the income generated by the stationary state X_s at each time step as long as the perturbation does not occur. $R_{agr}x_{agr_s}$ is the agricultural income when the land share $x_{agr,s}$ is allocated to agriculture. $\frac{(1-x_{agr,s})R_F}{F}$ is the income generated at each time step by a normal forest of rotation age F which land share is $(1-x_{agr,s})$.

V^e is a function such that $V^e(X) = V(D(X))$ for all X, then $\frac{\partial V^e}{\partial x_a}\Big|_{X_s} = \frac{\partial V(D)}{\partial x_a}\Big|_{X_s}$.

Karush-Kuhn-Tucker conditions state that at the optimum, the following four conditions 122, 123, 124 and 125 on the Lagrangian derivatives must be respected for all $a \geq 1$ and $t \geq 1$:

$$x^*_{agr,t}\frac{\partial L}{\partial x_{agr,t}} = 0 \tag{122}$$

$$x^*_{a,t}\frac{\partial L}{\partial x_{a,t}} = 0 \tag{123}$$

where $\{x^*_{a,t}\}$ are land shares at the optimum, and :

$$\frac{\partial L}{\partial x_{agr,t}} \leq 0 \tag{124}$$

$$\frac{\partial L}{\partial x_{a,t}} \leq 0 \tag{125}$$

As we assume that when $X_0 = X_s$, it is optimal that $X_1 = X_2 = ...X_- = ... = X_s$, then $x^*_{agr,t} > 0$ for all t, $x^*_{a,t} > 0$ for all t and for all $a \leq F$, and $x^*_{a,t} = 0$ for all t and for all $a > F$. Consequently, Karush-Kuhn-Tucker conditions translate as conditions 126, 127 and 128 : For $t \geq 1$

$$\frac{\partial L}{\partial x_{agr,t}}\Big|_{X_0=X_1=...=X_s,\{\mu_{a,t}\},\{\eta_t\}} = 0 \tag{126}$$

For $t \geq 1$ and $1 \leq a \leq F$:

$$\frac{\partial L}{\partial x_{a,t}}\Big|_{X_0=X_1=...=X_s,\{\mu_{a,t}\},\{\eta_t\}} = 0 \tag{127}$$

For $t \geq 1$ and $a > F$:

$$\frac{\partial L}{\partial x_{a,t}}\Big|_{X_0=X_1=...=X_s,\{\mu_{a,t}\},\{\eta_t\}} \leq 0 \tag{128}$$

By replacing the Lagrangian derivatives by their values, we finally obtain conditions 129, 130, 131 and 132 :

For $t \geq 1$:

$$\gamma^t(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_{agr} + \gamma^t \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha} \frac{\partial V^e}{\partial x_{agr,t}}\Big|_{X_s} - \eta_t = 0 \tag{129}$$

For $1 < a \leq F$ and $t \geq 1$:

$$\gamma^{t-1}(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(-R_{a-1}) + \gamma^t(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(+R_a)$$
$$+\gamma^t \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha} \frac{\partial V^e}{\partial x_{a,t}}\Big|_{X_s} + \mu_{a,t} - \mu_{a-1,t-1} - \eta_t = 0 \tag{130}$$

For $a > F$ and $t \geq 1$:

$$\gamma^{t-1}(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(-R_{a-1}) + \gamma^t(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(+R_a)$$
$$+\gamma^t \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha} \frac{\partial V^e}{\partial x_{a,t}}\Big|_{X_s} + \mu_{a,t} - \mu_{a-1,t-1} - \eta_t \leq 0 \tag{131}$$

For $a = 1$ and $t \geq 1$:

$$\gamma^t(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(+R_1) + \gamma^t \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha} \frac{\partial V^e}{\partial x_{1,t}}\Big|_{X_s} + \mu_{1,t} - \eta_t = 0 \tag{132}$$

The subscript t in terms $\frac{\partial V^e}{\partial x_{agr,t}}\Big|_{X_s}$ and $\frac{\partial V^e}{\partial x_{a,t}}\Big|_{X_s}$ is unnecessary, we can write $V^e(x_{1,t}, ..., x_{a,t}, ...) = V^e(x_1, ..., x_a, ...)$ and then $\frac{\partial V^e}{\partial x_{a,t}}\Big|_{X_s} = \frac{\partial V^e}{\partial x_a}\Big|_{X_s}$.

As demonstrated in Appendix 2, it is possible to combine conditions 129, 130, 131 and 132 to obtain following conditions 133, 134 and 135 :

$$\frac{\gamma}{1-\gamma}\left[(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_{agr} + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha} \frac{\partial V^e}{\partial x_{agr}}\Big|_{X_s}\right] - \sum_{t'=0}^{+\infty} \eta_{t'} = 0 \tag{133}$$

For $j \in \mathbb{N}$ and $j \leq F$:

$$\frac{1}{1-\gamma^j}\left[\gamma^j(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_j + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\left(\gamma\frac{\partial V^e}{\partial x_1}\Big|_{X_s} + ... + \gamma^j \frac{\partial V^e}{\partial x_j}\Big|_{X_s}\right)\right] + \sum_{k=1}^{+\infty} \mu_{j,kj} - \sum_{t'=0}^{+\infty} \eta_{t'} = 0 \tag{134}$$

For $j \in \mathbb{N}$ and $j > F$:

$$\frac{1}{1-\gamma^j}\left[\gamma^j(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_j + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\left(\gamma\frac{\partial V^e}{\partial x_1}\Big|_{X_s} + ... + \gamma^j \frac{\partial V^e}{\partial x_j}\Big|_{X_s}\right)\right] + \sum_{k=1}^{+\infty} \mu_{j,kj} - \sum_{t'=0}^{+\infty} \eta_{t'} \leq 0 \tag{135}$$

Karush-Kuhn-Tucker conditions also state that at the optimum, all dual variables must be positive, that is $\mu_{a,t} \geq 0$ for all $a \geq 1$ and all $t \geq 1$, and $\eta_t \geq 0$ for all $t \geq 1$. In addition, at the optimum, we must have $\mu_{a,t}(x_{a,t} - x_{a+1,t+1}) = 0$ for all $a \geq 1$ and $t \geq 1$ and $\eta_t(1 - x_{agr,t} - \sum_{a=1}^{+\infty} x_{a,t}) = 0$ for all $t \geq 1$. As, at the optimum, we have $x_{F+1,t} = 0 < \frac{1}{F} = x_{F,t}$ for all t, then $\mu_{F,t} = 0$ for all t and subsequently $\sum_{k=1}^{+\infty} \mu_{F,kF} = 0$.

As a consequence, from equation 134, we can write equation 136 :

$$\frac{1}{1-\gamma^F}\left[\gamma^F(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_F + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\left(\gamma\frac{\partial V^e}{\partial x_1}\Big|_{X_s} + ... + \gamma^F \frac{\partial V^e}{\partial x_F}\Big|_{X_s}\right)\right] - \sum_{t'=0}^{+\infty} \eta_{t'} = 0 \tag{136}$$

As $\sum_{k=1}^{+\infty} \mu_{j,kj} \geq 0$ for any $j \in \mathbb{N}$, the following inequality must hold :

$$\frac{1}{1-\gamma^F}\left[\gamma^F(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_F + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\left(\gamma\frac{\partial V^e(X_s)}{\partial x_1}\Big|_{X_s} + ... + \gamma^F \frac{\partial V^e}{\partial x_F}\Big|_{X_s}\right)\right]$$
$$\geq \frac{1}{1-\gamma^j}\left[\gamma^j(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_F + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\left(\gamma\frac{\partial V^e}{\partial x_1}\Big|_{X_s} + ... + \gamma^j \frac{\partial V^e}{\partial x_j}\Big|_{X_s}\right)\right] \tag{137}$$

Moreover, equation 133 and 136 show that :

$$\frac{\gamma}{1-\gamma}\left[(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_{agr} + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\frac{\partial V^e}{\partial x_{agr}}\Big|_{X_s}\right]$$
$$= \frac{1}{1-\gamma^F}\left[\gamma^F(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_F + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\right.$$
$$\left.\left(\gamma\frac{\partial V^e}{\partial x_1}\Big|_{X_s} + \gamma^2\frac{\partial V^e}{\partial x_2}\Big|_{X_s} + ... + \gamma^F \frac{\partial V^e}{\partial x_F}\Big|_{X_s}\right)\right] \tag{138}$$

Conditions 137 and 138 are the conditions we aimed at demonstrating. They are necessary conditions[4] for $X_1 = ... = X_t = ... = X_s$ to be optimal when $X_0 = X_s$.

4.4 Stationary land allocation and forest management : a numerical application

Equations 100 and 101 presented in section 4.3 help identify the determinants of stationary land allocation and forest management. However, they do not allow us to characterize the different types of stationary states nor do they allow us to assess the sensitivity of stationary states to preference parameters. The numerical approach used in this sections is aimed to answer these questions.

4. As the constraints are linear, the constraint qualifications are met for the program (see Leonard and Van Long (1992)) and Karush-Kuhn-Tucker conditions are indeed necessary.

4.4.1 A stochastic dynamic programming model

The model is analogous to the dynamic program introduced by the Bellman equation 97 and by constraints 98 and 99 in section 4.2 but a few differences are introduced.

The constraints are the same as in the analytical model except that, for ease of programming, the total land available is not normalized at 1 but at 100.

The income function remains the same (see equation 96) and the data on timber revenues (that is the sequence R_a) corresponds to the maritime pine forestry in southwestern France, it is taken from Couture and Reynaud (2011) and presented in table 4 :

Age-class	Age (in years)	Timber revenue (euros/ha)
1	5	0
2	10	11
3	15	29
4	20	97
5	25	390
6	30	1378
7	35	2917
8	40	4083

TABLE 4 – Timber revenue in function of the age

Assuming a 2%/year discount rate, Faustmann's age is 40 years. The agricultural income per land unit and per 5-year time step is 325. This value is *ad hoc* and is chosen so that the sum of discounted returns from agriculture is slightly smaller than the sum of deterministic discounted returns from the forest when it follows Faustmann's rule. Consequently, in the absence of risk, all land is allocated to forestry (see section 4.3) but the presence of a perturbation risk on the forest alone may affect this result, which is exactly what is investigated here.

The perturbation introduced in the numerical model is aimed to be a risk of storm. Several types of storms are specified and presented in table 5 below. The first type of storm (#1) destroys completely the age-class 8 (the oldest possible one), the second type (#2) destroys completely age-classes 7 and 8 (the two oldest ones), and so on. This representation of risk reflects the fact that the vulnerability of the forest increases over its age, which is a fundamental characteristic of the risk of storm on forests (see Schelhaas et al. (2010)). Agriculture is assumed not to be affected by storms.

#event (e)	Destroyed age classes (d_e)	Prob of occurrence (p_e)
0	None	0.933
1	8	0.016
2	7+8	0.014
3	6+7+8	0.012
4	5+6+7+8	0.009
5	4+5+6+7+8	0.007
6	3+4+5+6+7+8	0.005
7	2+3+4+5+6+7+8	0.003
8	All	0.001

TABLE 5 – Storms and probabilities

The Bellman equation associated with this program is given in equation 139 :

$$V_t = \max_{X_{t+1}} \left\{ \left((1-\beta)\Pi(X_t, X_{t+1})^{\frac{\sigma-1}{\sigma}} + \beta[\sum_{e=0}^{10} p_e V(d_e(X_{t+1}))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}} \right)^{\frac{\sigma}{\sigma-1}} \right\} \tag{139}$$

$X_{t+1} = (x_{agr,t+1}, x_{1,t+1}; x_{2,t+1}; ...; x_{a,t+1}; ...)$ are the land shares that are realized at $t+1$ in the absence of perturbation between t and $t+1$ (see Figure 2), d_e are the functions that describe the consequences on these decided land shares of the different events e presented in table 2, and p_e their associated probabilities.

The program is solved using a value function iteration algorithm (see Judd (1998)) that is computed using the GAMS language and solvers. This algorithm supposes an infinite time horizon, which is necessary as the focus is put on stationary states. The convergence of the algorithm is ensured by the Contraction Mapping Theorem (see Judd (1998)). In addition, within the algorithm, the value function is given a third order polynomial form :

$$V(X) = \alpha_{agr} \cdot x_{agr} + \sum_{i=1}^{10} \sum_{j=i}^{10} \sum_{k=j}^{10} \alpha_{i,j,k} \cdot x_i \cdot x_j \cdot x_k + \sum_{l=1}^{10} \sum_{m=l}^{10} \alpha_{l,m} \cdot x_l \cdot x_m$$

$$+ \sum_{n=1}^{10} \alpha_n \cdot x_n + \sum_{o=1}^{10} \sum_{o=p}^{10} \alpha_{agr,o,p} \cdot x_{agr} \cdot x_o \cdot x_p + \sum_{q=1}^{10} \alpha_{agr,q} \cdot x_{agr} \cdot x_q \quad (140)$$

The algorithm is run for a given set of exogenous parameters and determines the corresponding value function. Once the value function is known, an optimal sequence of decisions can be computed on the basis of the Bellman equation, starting from bare land, and a stationary state is given as the long-term steady state.

4.4.2 Results

The results presented below reveal the existence of two types of stationary states : i) periodic forests following Faustmann's rule, and ii) mixed land allocations between agriculture and a normal forest following Faustmann's rule.

The existence of stationary periodic forests in a deterministic context is well documented (see for example Salo and Tahvonen (2002b)) and is shown to be related to the discrete time setting. Although it is not formally demonstrated here, the same reason probably applies in a stochastic context.

The fact that mixed land allocations can be stationary in a risky context is permitted by the endogeneity of condition 101 (see section 4.3), which is not a limit condition on exogenous parameters as in the deterministic case.

In addition, the absence of periodicity in the forest management when land allocation is mixed also seems to be related to condition 101. Indeed, it consists of an equality constraint that depends on land variables that cannot generally be verified when land variables are not constant.

The fact that stationary mixed land allocations are constant over time allows us to associate a single land allocation to a given set of exogenous parameters. Thus, the role of preferences on land allocation is assessed through sensitivity analyses. The role of risk aversion (parameter α) is presented in Figure 17 ($\sigma = 100$ so that intertemporal preferences are very low) :

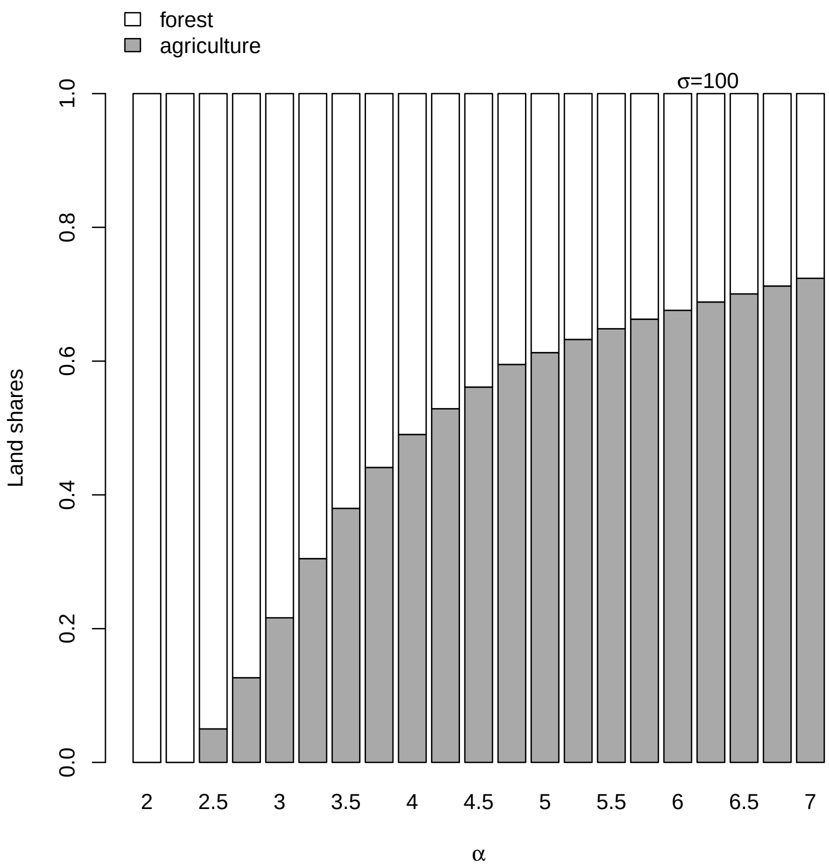

FIGURE 17 – Impact of risk aversion on land allocation

Figure 17 shows that land allocation responds gradually to risk aversion and that there exists a continuum of mixed land allocations. Moreover, the fact that risk aversion fosters agriculture reveals that mixed land allocation are used by the landowner as a diversification strategy against the risk of perturbation.

A similar figure is obtained, although with higher agricultural land shares, when the destruction of the forest is total in case any storm described in Table 5 occurs. The existence of mixed land allocations under these circumstances confirms the specificity of recursive preferences in comparison to expected utility preferences for which mixed land allocations do not exist in presence of a risk of total destruction (see section 4.3.1 for an analytical discussion on this matter).

The role of intertemporal preferences is presented in Figure 18 (the smaller σ is, the less flexible intertemporal substitutions are) :

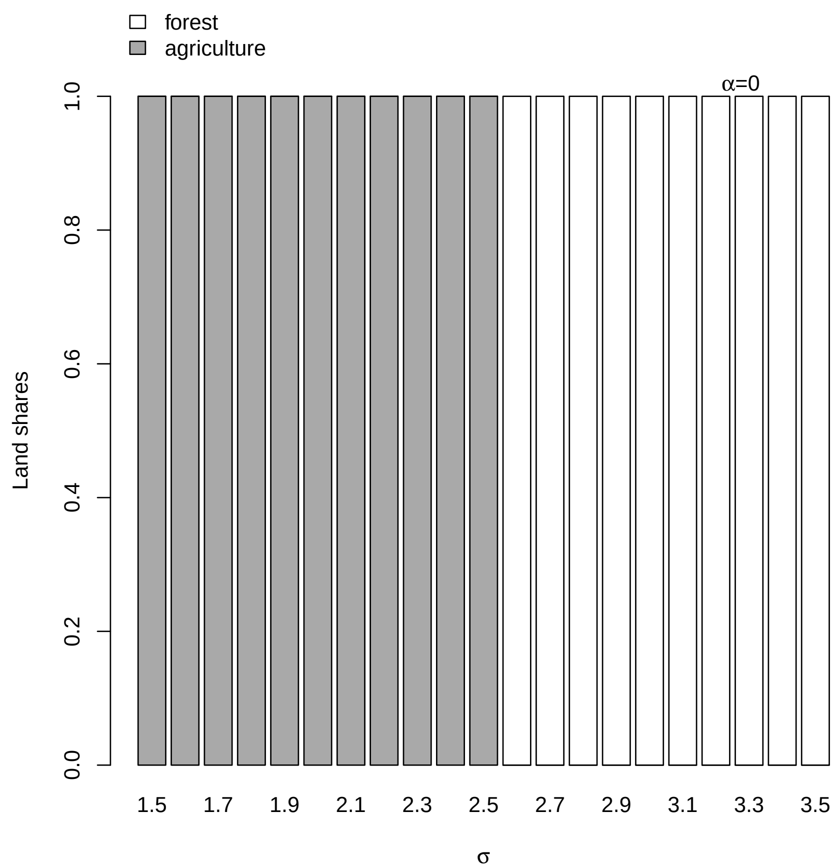

FIGURE 18 – Impact of intertemporal preferences on land allocation

Unlike risk aversion, the impact of intertemporal preferences on land allocation seems to be discontinuous with a threshold value for σ separating pure agriculture and pure forestry land allocations. Apart from this discontinuity property, intertemporal preferences also seem to foster agriculture. As agriculture is not subject to the risk of storm, allocating land to agriculture is indeed a precautionary behavior that avoids future incomes to be irregular over time in case a storm occurs.

The joint influence of both types of preferences is shown on Figure 19 :

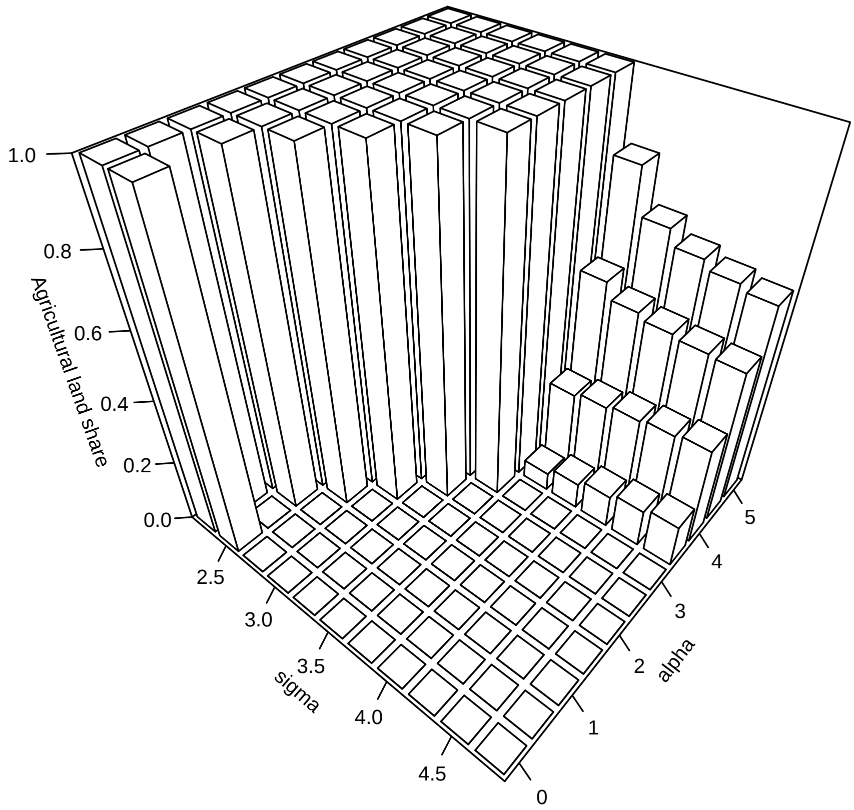

Figure 19 – Joint impact of preferences on land allocation

Figure 19 reveals that the "threshold effect" linked to intertemporal preferences (σ) is still observable when the landowner is also risk-averse. However, the threshold value for σ is dependent on risk aversion (α). In fact, there seems to be a substitutability between risk aversion and intertemporal preferences, at least for relatively low levels of risk aversion. However, for higher levels of risk aversion, the threshold value seems to be stabilized but separates pure agriculture and mixed land allocations.

In addition, within these mixed land allocations, intertemporal preferences seem to accentuate the effect of risk aversion. The elasticity of the agricultural land share to risk aversion is for example larger for $\sigma = 3.75$ than for $\sigma = 4.75$.

In any case, Figure 19 shows that the respective roles of risk aversion and intertemporal preferences on stationary land allocation are different and that there are interaction effects between these two types of preferences. This is a clear demonstration that recursive preferences are crucial to understand the role of preferences on allocating land between agriculture and forestry, or between any two sectors of which at least one is dynamic.

With regard to forest management, in all the simulations carried out to draw Figures 17, 18 and 19, the forest is always shown to follow Faustmann's rule, that is to say the rotation age is unique and is 40 years. However, when an alternative land use is not possible, forest management does not necessarily follows Faustmann's rule and is affected by preferences. More precisely, the stationary rotation age may not be unique and may be lower than Faustmann's age. This reduction in rotation ages reflects a precautionary behavior. In conclusion, precautionary behaviors can be expressed through both land allocation and forest management but in the context of our numerical model, land allocation is consistently preferred by the landowner.

4.5 Discussion

The first important result of this study is that land allocation between agriculture and a multiple age-class forest that is subject to a risk of perturbation depends on the risk and on preferences. More precisely, it is shown that to a given set of values for exogenous parameters (including the probability of the risk and preference parameters) corresponds a single stationary land allocation. Most importantly, land allocation is revealed to be used as a means of diversification against risk by the landowner.

The other important result is that land allocation and forest management are interdependent. Precautionary behaviors can indeed be adopted through these two levers and it is shown that in the context of our numerical model, the land allocation lever when available is preferred over the forest management lever.

This study extends the deterministic approach presented in Salo and Tahvonen (2004) to a stochastic setting, and demonstrates and clarifies the joint role of risk and preferences in allocating land. In parallel, it also extends studies such as Parks (1995) that are based on stochastic models of land allocation that do not take into account the dynamic nature of forestry.

This study also shows that recursive preferences are essential in the understanding of land allocation between a dynamic activity such as forestry and an alternative use. Risk aversion and intertemporal preferences are indeed shown to have different effects on land allocation and should therefore be distinguished.

4.6 Appendices

Appendix 1

Let us recall equations 113 given in section 4.3.2 :

$$U_0(X_0, X_1, ..., X_t, ...) = \left((1-\beta)\Pi(X_0, X_1)^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_1^{1-\alpha} + pV(D(X_1))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}}\right)^{\frac{\sigma}{\sigma-1}} \tag{141}$$

$$U_1(X_1, X_2, ..., X_t, ...) = \left((1-\beta)\Pi(X_1, X_2)^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_2^{1-\alpha} + pV(D(X_2))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}}\right)^{\frac{\sigma}{\sigma-1}} \tag{142}$$

[...]

$$U_{t-1}(X_{t-1}, X_t, ...) = \left((1-\beta)\Pi(X_{t-1}, X_t)^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_t^{1-\alpha} + pV(D(X_t))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}}\right)^{\frac{\sigma}{\sigma-1}} \tag{143}$$

$$U_t(X_t, X_{t+1}, ...) = \left((1-\beta)\Pi(X_t, X_{t+1})^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_{t+1}^{1-\alpha} + pV(D(X_{t+1}))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}}\right)^{\frac{\sigma}{\sigma-1}} \tag{144}$$

[...]

From these equations, using the chain rule, we know that :

$$\frac{\partial U_0}{\partial x_{agr,t}} = \frac{\partial U_0}{\partial U_1}\frac{\partial U_1}{\partial U_2}...\frac{\partial U_{t-1}}{\partial x_{agr,t}} \tag{145}$$

$$\frac{\partial U_0}{\partial x_{a,t}} = \frac{\partial U_0}{\partial U_1}\frac{\partial U_1}{\partial U_2}...\frac{\partial U_{t-1}}{\partial x_{a,t}} \tag{146}$$

In addition, by deriving equation 143, we can write :

$$\frac{\partial U_{t-1}}{\partial x_{agr,t}} = \frac{\sigma}{\sigma-1}\left((1-\beta)\Pi(X_{t-1},X_t)^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_t^{1-\alpha} + pV(D(X_t))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}}\right)^{\frac{\sigma}{\sigma-1}-1}$$

$$\left[(1-\beta)(\frac{\sigma-1}{\sigma})\Pi(X_{t-1},X_t)^{\frac{\sigma-1}{\sigma}-1}\frac{\partial\Pi(X_{t-1},X_t)}{\partial x_{agr,t}}\right.$$

$$+\beta\frac{\frac{\sigma-1}{\sigma}}{1-\alpha} * [(1-p)U_t^{1-\alpha} + pV(D(X_t))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}-1}$$

$$\left.\left((1-\alpha)(1-p)U_t^{-\alpha}\frac{\partial U_t}{\partial x_{agr,t}} + p(1-\alpha)V(D(X_t))^{-\alpha}\frac{\partial V(D(X_t))}{\partial x_{agr,t}}\right)\right] \tag{147}$$

$$\frac{\partial U_{t-1}}{\partial x_{a,t}} = \frac{\sigma}{\sigma-1}\left((1-\beta)\Pi(X_{t-1},X_t)^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_t^{1-\alpha} + pV(D(X_t))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}}\right)^{\frac{\sigma}{\sigma-1}-1}$$

$$\left[(1-\beta)(\frac{\sigma-1}{\sigma})\Pi(X_{t-1},X_t)^{\frac{\sigma-1}{\sigma}-1}\frac{\partial\Pi(X_{t-1},X_t)}{\partial x_{a,t}}\right.$$

$$+\beta\frac{\frac{\sigma-1}{\sigma}}{1-\alpha} * [(1-p)U_t^{1-\alpha} + pV(D(X_t))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}-1}$$

$$\left.\left((1-\alpha)(1-p)U_t^{-\alpha}\frac{\partial U_t}{\partial x_{a,t}} + p(1-\alpha)V(D(X_t))^{-\alpha}\frac{\partial V(D(X_t))}{\partial x_{a,t}}\right)\right] \tag{148}$$

By deriving equation 144, we can also write :

$$\frac{\partial U_t}{\partial x_{agr,t}} = \frac{\sigma}{\sigma-1}\left((1-\beta)\Pi(X_t,X_{t+1})^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_{t+1}^{1-\alpha} + pV(D(X_{t+1}))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}}\right)^{\frac{\sigma}{\sigma-1}-1}$$

$$(1-\beta)(\frac{\sigma-1}{\sigma})\Pi(X_t,X_{t+1})^{\frac{\sigma-1}{\sigma}-1}\frac{\partial\Pi(X_t,X_{t+1})}{\partial x_{agr,t}} \tag{149}$$

$$\frac{\partial U_t}{\partial x_{a,t}} = \frac{\sigma}{\sigma-1}\left((1-\beta)\Pi(X_t,X_{t+1})^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_{t+1}^{1-\alpha} + pV(D(X_{t+1}))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}}\right)^{\frac{\sigma}{\sigma-1}-1}$$

$$(1-\beta)(\frac{\sigma-1}{\sigma})\Pi(X_t,X_{t+1})^{\frac{\sigma-1}{\sigma}-1}\frac{\partial\Pi(X_t,X_{t+1})}{\partial x_{a,t}} \tag{150}$$

In addition, after equation 96 (see section 4.2), we know that $\frac{\partial\Pi(X_{t-1},X_t)}{\partial x_{a,t}} = -R_{a-1}$ (for $a > 1$ and 0 otherwise), $\frac{\partial\Pi(X_t,X_{t+1})}{\partial x_{a,t}} = R_a$, $\frac{\partial\Pi(X_{t-1},X_t)}{\partial x_{agr,t}} = 0$ and $\frac{\partial\Pi(X_t,X_{t+1})}{\partial x_{agr,t}} = R_{agr}$.

Moreover, we define a function V^e such that $V^e(X) = V(D(X))$ for all X, then we can write $\frac{\partial V(D(X_t))}{\partial x_{agr,t}} = \frac{\partial V^e(X_t)}{\partial x_{agr,t}}$ and $\frac{\partial V(D(X_t))}{\partial x_{a,t}} = \frac{\partial V^e(X_t)}{\partial x_{a,t}}$.

Then, we want to evaluate $\frac{\partial U_{t-1}}{\partial x_{agr,t}}$ and $\frac{\partial U_{t-1}}{\partial x_{a,t}}$ when $X_{t-1} = X_t = ... = X_s$ where $X_s = (x_{agr,s}, x_{1,s}, ..., x_{a,s}, ...)$ describes a mixed land allocation between agriculture and a normal forest of rotation age F (see section 4.3.2). Considering this, the income provided by the land system in the stationary state X_s as long as no perturbation occurs is constant over time and we can write : $\Pi X_t, X_{t+1} = \Pi(X_s, X_s) = \Pi_s = R_{agr} x_{agr,s} + (1 - x_{agr,s}) \frac{R_F}{F}$ for all t.

In addition, we assume that when the land system is in the state X_s at a given time t, it is optimal to remain in this state as long as the perturbation does not occur. This means that when $X_t = X_s$ (which is constrained from a time t standpoint), decisions $X_{t+1} = X_{t+2} = ... = X_s$ are optimal, in the sense that U_t but also $U_{t+1}, U_{t+2}, ...$, as defined by equations 144,... are maximized under constraints 98 and 99. We then have $\max_{X_{t+1}, X_{t+2},...} \{U_t\} = \max_{X_{t+2}, X_{t+3}...} \{U_{t+1}\} = ... = V(X_s)$.

As the landowner faces the same program at every time step, this assumption holds for all t. In particular, if $X_0 = X_s$ (constrained), then decisions $X_1 = X_2 = ... = X_t = ... = X_s$ are optimal and $\max_{X_1, X_2,...} \{U_0\} = \max_{X_2, X_3...} \{U_1\} = ... = V(X_s)$.

Under these conditions, at $X_{t-1} = X_t = ... = X_s$, the derivative $\frac{\partial U_{t-1}}{\partial x_{agr,t}}$ and $\frac{\partial U_{t-1}}{\partial x_{a,t}}$ takes the following value :

$$\frac{\partial U_{t-1}}{\partial x_{agr,t}}\Big|_{X_{t-1}=X_t=\ldots=X_s} = \frac{\sigma}{\sigma-1}\Big(V(X_s)^{\frac{\sigma-1}{\sigma}}\Big)^{\frac{\sigma}{\sigma-1}-1}$$

$$\Big[\beta\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}[(1-p)V(X_s)^{1-\alpha}+pV^e(X_s)^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}-1}$$

$$\Big((1-\alpha)(1-p)V(X_s)^{-\alpha}\frac{\sigma}{\sigma-1}\Big(V(X_s)^{\frac{\sigma-1}{\sigma}}\Big)^{\frac{\sigma}{\sigma-1}-1}(1-\beta)\frac{\sigma-1}{\sigma}\Pi_s^{\frac{\sigma-1}{\sigma}-1}(R_{agr})$$

$$+p(1-\alpha)V^e(X_s)^{-\alpha}\frac{\partial V^e}{\partial x_{agr,t}}\Big|_{X_s}\Big)\Big]$$

$$= \beta(1-p)\Big(\frac{[(1-p)V(X_s)^{1-\alpha}+pV^e(X_s)^{1-\alpha}]^{\frac{1}{1-\alpha}}}{V(X_s)}\Big)^{\alpha-\frac{1}{\sigma}}$$

$$\Big[(1-\beta)\Big(\frac{V(X_s)}{\Pi_s}\Big)^{\frac{1}{\sigma}}R_{agr}+\frac{p}{1-p}\Big(\frac{V(X_s)}{V^e(X_s)}\Big)^{\alpha}\frac{\partial V^e}{\partial x_{agr,t}}\Big|_{X_s}\Big] \quad (151)$$

$$\frac{\partial U_{t-1}}{\partial x_{a,t}}\Big|_{X_{t-1}=X_t=\ldots=X_s} = \frac{\sigma}{\sigma-1}\Big(V(X_s)^{\frac{\sigma-1}{\sigma}}\Big)^{\frac{\sigma}{\sigma-1}-1}$$

$$\Big[(1-\beta)(\frac{\sigma-1}{\sigma})\Pi_s^{\frac{\sigma-1}{\sigma}-1}(-R_{a-1})$$

$$+\beta\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}[(1-p)V(X_s)^{1-\alpha}+pV^e(X_s)^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}-1}$$

$$\Big((1-\alpha)(1-p)V(X_s)^{-\alpha}\frac{\sigma}{\sigma-1}\Big(V(X_s)^{\frac{\sigma-1}{\sigma}}\Big)^{\frac{\sigma}{\sigma-1}-1}(1-\beta)\frac{\sigma-1}{\sigma}\Pi_s^{\frac{\sigma-1}{\sigma}-1}(R_a)$$

$$+p(1-\alpha)V^e(X_s)^{-\alpha}\frac{\partial V^e}{\partial x_{a,t}}\Big|_{X_s}\Big)\Big]$$

$$= (1-\beta)\Big(\frac{V(X_s)}{\Pi_s}\Big)^{\frac{1}{\sigma}}(-R_{a-1})$$

$$+\beta(1-p)\Big(\frac{[(1-p)V(X_s)^{1-\alpha}+pV^e(X_s)^{1-\alpha}]^{\frac{1}{1-\alpha}}}{V(X_s)}\Big)^{\alpha-\frac{1}{\sigma}}$$

$$\Big[(1-\beta)\Big(\frac{V(X_s)}{\Pi_s}\Big)^{\frac{1}{\sigma}}R_a+\frac{p}{1-p}\Big(\frac{V(X_s)}{V^e(X_s)}\Big)^{\alpha}\frac{\partial V^e}{\partial x_{a,t}}\Big|_{X_s}\Big] \quad (152)$$

We then define γ as follows :

$$\gamma = \beta(1-p)\left(\frac{[(1-p)V(X_s)^{1-\alpha} + pV^e(X_s)^{1-\alpha}]^{\frac{1}{1-\alpha}}}{V(X_s)}\right)^{\alpha-\frac{1}{\sigma}} \tag{153}$$

At last, we obtain :

$$\frac{\partial U_{t-1}}{\partial x_{agr,t}}\Big|_{X_{t-1}=X_t=...=X_s} = \gamma\left[(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_{agr} + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\frac{\partial V^e}{\partial x_{agr,t}}\Big|_{X_s}\right] \tag{154}$$

$$\frac{\partial U_{t-1}}{\partial x_{a,t}}\Big|_{X_{t-1}=X_t=...=X_s}$$

$$= (1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(-R_{a-1}) + \gamma\left[(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_a + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\frac{\partial V^e}{\partial x_{a,t}}\Big|_{X_s}\right] \tag{155}$$

Then, we want to express derivatives $\frac{\partial U_{t-2}}{\partial x_{agr,t}}$ and $\frac{\partial U_{t-2}}{\partial x_{a,t}}$. Using the chain rule, we can write them as follows :

$$\frac{\partial U_{t-2}}{\partial x_{agr,t}} = \frac{\partial U_{t-2}}{\partial U_{t-1}}\frac{\partial U_{t-1}}{\partial x_{agr,t}}$$

$$= \frac{\sigma}{\sigma-1}\left((1-\beta)\Pi(X_{t-2},X_{t-1})^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_{t-1}^{1-\alpha} + pV(D(X_{t-1}))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}}\right)^{\frac{\sigma}{\sigma-1}-1}$$

$$\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}\beta[(1-p)U_{t-1}^{1-\alpha} + pV(D(X_{t-1}))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}-1}(1-\alpha)(1-p)U_{t-1}^{-\alpha}\frac{\partial U_{t-1}}{\partial x_{agr,t}} \tag{156}$$

$$\frac{\partial U_{t-2}}{\partial x_{a,t}} = \frac{\partial U_{t-2}}{\partial U_{t-1}}\frac{\partial U_{t-1}}{\partial x_{a,t}}$$

$$= \frac{\sigma}{\sigma-1}\left((1-\beta)\Pi(X_{t-2},X_{t-1})^{\frac{\sigma-1}{\sigma}} + \beta[(1-p)U_{t-1}^{1-\alpha} + pV(D(X_{t-1}))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}}\right)^{\frac{\sigma}{\sigma-1}-1}$$

$$\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}\beta[(1-p)U_{t-1}^{1-\alpha} + pV(D(X_{t-1}))^{1-\alpha}]^{\frac{\frac{\sigma-1}{\sigma}}{1-\alpha}-1}(1-\alpha)(1-p)U_{t-1}^{-\alpha}\frac{\partial U_{t-1}}{\partial x_{a,t}} \tag{157}$$

We assume that $X_{t-2} = X_s$. As assumed above, decisions $X_{t-1} = X_t = ... = X_s$ are then optimal and we have $\max\limits_{X_{t-1},X_t,...}\{U_{t-2}\} = V(X_s)$.

Therefore, at $X_{t-2} = X_{t-1} = X_t = ... = X_s$, $\frac{\partial U_{t-2}}{\partial x_{agr,t}}$ and $\frac{\partial U_{t-2}}{\partial x_{a,t}}$ take the following values :

$$\frac{\partial U_{t-2}}{\partial x_{agr,t}}\Big|_{X_{t-2}=X_{t-1}=...=X_s}$$

$$= \beta(1-p)\left(\frac{[(1-p)V(X_s)^{1-\alpha} + pV^e(X_s)^{1-\alpha}]^{\frac{1}{1-\alpha}}}{V(X_s)}\right)^{\alpha-\frac{1}{\sigma}} \frac{\partial U_{t-1}}{\partial x_{agr,t}}\Big|_{X_{t-1}=X_t=...=X_s}$$

$$= \gamma \frac{\partial U_{t-1}}{\partial x_{agr,t}}\Big|_{X_{t-1}=X_t=...=X_s}$$

$$= \gamma\left(\gamma(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_{agr} + \gamma\frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha} \frac{\partial V^e}{\partial x_{agr,t}}\Big|_{X_s}\right) \quad (158)$$

$$\frac{\partial U_{t-2}}{\partial x_{a,t}}\Big|_{X_{t-2}=X_{t-1}=...=X_s}$$

$$= \beta(1-p)\left(\frac{[(1-p)V(X_s)^{1-\alpha} + pV^e(X_s)^{1-\alpha}]^{\frac{1}{1-\alpha}}}{V(X_s)}\right)^{\alpha-\frac{1}{\sigma}} \frac{\partial U_{t-1}}{\partial x_{a,t}}\Big|_{X_{t-1}=X_t=...=X_s}$$

$$= \gamma \frac{\partial U_{t-1}}{\partial x_{a,t}}\Big|_{X_{t-1}=X_t=...=X_s}$$

$$= \gamma\left((1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(-R_{a-1}) + \gamma(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_a\right.$$

$$\left. + \gamma\frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha} \frac{\partial V^e}{\partial x_{a,t}}\Big|_{X_s}\right) \quad (159)$$

By repeating the same rationale recursively and by assuming that $X_0 = X_s$, we show that :

$$\frac{\partial U_0}{\partial x_{agr,t}}\Big|_{X_0=X_1=...=X_s} = \gamma^t(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_{agr}$$

$$+ \gamma^t\frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha} \frac{\partial V^e}{\partial x_{agr,t}}\Big|_{X_s} \quad (160)$$

$$\frac{\partial U_0}{\partial x_{a,t}}\Big|_{X_0=X_1=...=X_s}$$

$$= \gamma^{t-1}\Bigg((1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(-R_{a-1}) + \gamma(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_a$$

$$+ \gamma\frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\frac{\partial V^e}{\partial x_{a,t}}\Big|_{X_s}\Bigg) \quad (161)$$

Expression 161 holds for $a > 1$, when $a = 1$ we have $\frac{\partial \Pi(X_{t-1},X_t)}{\partial x_{1,t}} = 0$ for all t as mentioned above and expression 161 becomes :

$$\frac{\partial U_0}{\partial x_{1,t}}\Big|_{X_0=X_1=...=X_s} = \gamma^t(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_1 + \gamma^t\frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\frac{\partial V^e}{\partial x_{1,t}}\Big|_{X_s} \quad (162)$$

These expressions would actually hold true for all stationary states X^* (that are states which are optimal to carry on as long as no perturbation occurs) and not only for state X_s.

Appendix 2

- Demonstration of :

  For $j \leq F$:

$$\frac{1}{1-\gamma^j}\left[(1-\beta)\gamma^j\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_j + \frac{p}{1-p}\left(\frac{V(X_s)}{V(X_s)^e}\right)^{\alpha}\left(\gamma\frac{\partial V^e}{\partial x_1}\Big|_{X_s} + ... + \gamma^j\frac{\partial V^e}{\partial x_j}\Big|_{X_s}\right)\right]$$

$$+ \sum_{k=1}^{+\infty}\mu_{j,kj} - \sum_{t'=0}^{+\infty}\eta_{t'} = 0 \quad (163)$$

  We start from conditions 164 and 165 below (see section 4.3.2) : For $1 < a \leq F$ and $t \geq 1$:

$$C_{a,t} = \gamma^{t-1}(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(-R_{a-1}) + \gamma^t(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(+R_a)$$

$$+ \gamma^t\frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\frac{\partial V^e}{\partial x_a}\Big|_{X_s} + \mu_{a,t} - \mu_{a-1,t-1} - \eta_t = 0 \quad (164)$$

For $t \geq 1$:

$$C_{1,t} = \gamma^t(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(+R_1) + \gamma^t \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha} \frac{\partial V^e}{\partial x_1}\Big|_{X_s} + \mu_{1,t} - \eta_t = 0 \quad (165)$$

Then, for any $j \leq F$ and any $t \geq j$, we sum the equations $C_{a,t'} = 0$ over $\{(a,t')\} \in \{(1+k, t-j+1+k) | k \in [0; j-1]\}$. We obtain the following equation :

$$\gamma^t(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_j + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha} \gamma^{t-j}\left(\gamma \frac{\partial V^e(X_s)}{\partial x_1}\Big|_{X_s} + ... + \gamma^j \frac{\partial V^e(X_s)}{\partial x_j}\Big|_{X_s}\right)$$
$$+ \mu_{j,t} - \sum_{t'=t-j+1}^{t} \eta_{t'} = 0 \quad (166)$$

We then sum this equation over $t \in \{kj | k \in \mathbb{N}^*\}$ to obtain :

$$\frac{1}{1-\gamma^j}\left[(1-\beta)\gamma^j\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_j + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\left(\gamma \frac{\partial V^e}{\partial x_1}\Big|_{X_s} + ... + \gamma^j \frac{\partial V^e}{\partial x_j}\Big|_{X_s}\right)\right]$$
$$+ \sum_{k=1}^{+\infty} \mu_{j,kj} - \sum_{t'=0}^{+\infty} \eta_{t'} = 0 \quad (167)$$

The last sum can be written only if the geometric series $\{\gamma^{kj}\}_{k\in\mathbb{N}}$ is convergent. This is the case when $\gamma < 1$, which we assume.

- Demonstration of :
  For $j > F$:

$$\frac{1}{1-\gamma^j}\left[(1-\beta)\gamma^j\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}} R_j + \frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\left(\gamma \frac{\partial V^e}{\partial x_1}\Big|_{X_s} + ... + \gamma^j \frac{\partial V^e}{\partial x_j}\Big|_{X_s}\right)\right]$$
$$+ \sum_{k=1}^{+\infty} \mu_{j,kj} - \sum_{t'=0}^{+\infty} \eta_{t'} \leq 0 \quad (168)$$

The demonstration is similar, except that for $a > F$ and for all t, we have the inequality

169 :

$$\gamma^{t-1}(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(-R_{a-1})+\gamma^{t}(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}(+R_{a})$$
$$+\gamma^{t}\frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\frac{\partial V^e}{\partial x_a}\Big|_{X_s}+\mu_{a,t}-\mu_{a-1,t-1}-\eta_t\leq 0 \quad (169)$$

Hence the inequality 168.

– Demonstration of :

$$\frac{\gamma}{1-\gamma}\left[(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}R_{agr}+\frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\frac{\partial V^e}{\partial x_{agr}}\Big|_{X_s}\right]-\sum_{t'=0}^{+\infty}\eta_{t'}=0 \quad (170)$$

We start from equation 171 below (see section 4.3.2) :

$$\gamma^{t}(1-\beta)\left(\frac{V(X_s)}{\Pi_s}\right)^{\frac{1}{\sigma}}R_{agr}+\gamma^{t}\frac{p}{1-p}\left(\frac{V(X_s)}{V^e(X_s)}\right)^{\alpha}\frac{\partial V^e}{\partial x_{agr}}\Big|_{X_s}-\eta_t=0 \quad (171)$$

We sum these equations over $t\in\mathbb{N}^*$ and we obtain equation 170 (with $\gamma<1$).

Conclusion générale

Synthèse des résultats

Les travaux réalisés dans le cadre de cette thèse portent sur la sylviculture en futaie régulière d'une forêt à plusieurs classes d'âge en présence d'un risque de tempête.

Le résultat le plus important issu de ces travaux est que plusieurs âges de rotation peuvent coexister à l'état stationnaire et qu'ils dépendent dans le cas général du risque et des préférences, que ce soit de l'aversion au risque (préférence pour un revenu sûr) ou des préférences intertemporelles (préférence pour un revenu régulier). Lorsque l'âge de rotation est unique, la forêt est périodiquement stationnaire, en lien avec les résultats en contexte déterministe proposés par Salo and Tahvonen (2002b). En revanche, lorsque plusieurs âges de rotation coexistent, la forêt présente toujours une structure en classes d'âge qui est constante au cours du temps. Dans ce cas, il est possible de calculer un âge de rotation moyen pondéré par les surfaces de chaque classe d'âge respectivement récoltées à chaque période.

L'existence de plusieurs âges de récolte à l'état stationnaire fait apparaître des forêts à la structure graduelle dans lesquelles la surface occupée par une classe donnée diminue avec son âge. Ce type de gestion peut être justifié en contexte de risque et en présence de préférences car elle permet de réduire l'exposition et la vulnérabilité au risque, tout en réduisant le moins possible le revenu forestier.

En outre, comme dans le cas déterministe (voir Salo and Tahvonen (2002b)), le ou les âges de rotation dépendent également des dynamiques de croissance biologique de la forêt, des prix et du taux d'actualisation mais pas de l'état initial de la forêt. Ainsi, pour un type de sylviculture donné et dans un contexte économique donné, il est possible d'associer un âge de rotation stationnaire, unique ou moyen, à un niveau de risque et de préférences donné.

Il apparaît que l'âge moyen de rotation à l'état stationnaire décroît lorsque l'aversion au risque ou les préférences intertemporelles se renforcent. Ce résultat révèle que l'âge de rotation est un levier majeur dans l'adoption de comportements de précaution en réponse au risque de tempête et aux préférences. Par ailleurs, la possibilité de répartir les prélèvements de bois entre plusieurs âges de récolte permet à l'âge moyen de rotation de répondre de manière graduelle (ou continue) à des variations des paramètres de risque et de préférences, bien que l'ensemble des âges de récolte possibles soit discret. Ces résultats constituent une extension dans un cadre à plusieurs classes d'âge des conclusions obtenues sur la base de modèles de rotation à classe d'âge unique (modèle de Faustmann).

Un deuxième résultat important est qu'en parallèle des préférences, la composante probabiliste du risque (*hazard*) a également une influence majeure sur la gestion forestière. Une analyse de statique comparative montre qu'un risque de tempête plus important entraîne un comportement plus précautionneux et par voie de conséquence un âge moyen de rotation plus faible.

L'existence d'anticipations dynamiques sur un risque de tempête dont la probabilité est à la hausse entraîne également un comportement de précaution et une baisse de l'âge de rotation. Toutefois, l'impact de ces anticipations est faible voire même négligeable lorsque les préférences sont faibles. Ce résultat vaut au moins pour la sylviculture du pin maritime (considérée dans le modèle) pour laquelle les âges de rotation sont faibles au regard des dynamiques de changement climatique.

En outre, l'effet des anticipations dépend des préférences. L'aversion au risque contribue à réduire l'âge de rotation mais très faiblement et la composante d'incertitude du changement climatique n'a pas dans ce cas d'effet additionnel par rapport à sa composante tendancielle. En revanche, il existe un effet significatif des anticipations lorsque les préférences intertemporelles sont fortes. Dans ce cas, l'effet du changement climatique est double, d'une part l'augmentation tendancielle du risque de tempête raccourcit les rotations, d'autre part l'incertitude quant au changement climatique les allonge.

Enfin, un troisième et dernier résultat important est que lorsqu'un usage alternatif des terres sans risque et offrant un rendement régulier est possible, l'allocation des terres entre la forêt et cet usage alternatif est potentiellement mixte lorsque le propriétaire terrien a des préférences. Dans ce cas, l'allocation dépend du risque et des préférences. Plus précisément, l'aversion au risque et les préférences intertemporelles favorisent toutes deux l'allocation des terres à l'usage alternatif mais leurs effets respectifs sont clairement distincts. L'allocation des terres répond en effet de manière graduelle à l'aversion au risque alors qu'il existe un effet de seuil pour les préférences intertemporelles.

L'existence d'allocations mixtes des terres en contexte de risque détonne avec les solutions en coin obtenues en contexte déterministe. Cette diversification est un comportement de précaution qui permet de réduire l'exposition et la vulnérabilité globales du système au risque en allouant une partie des terres à l'usage alternatif non risqué, bien que le rendement (certain) de cet usage alternatif soit inférieur au rendement espéré de la forêt. Ces résultats sont une illustration de la théorie du portefeuille (voir Markowitz (1952)).

Cependant, dans le cas présent, la diversification de l'usage des terres n'est qu'un seul des deux leviers permettant de s'adapter au risque, l'autre étant la gestion forestière. Il est montré que ces deux leviers sont interdépendants. En particulier, il est montré dans le contexte des modèles numériques utilisés dans cette thèse, que lorsqu'il est disponible, seul le levier de l'allocation des terres est utilisé et que dans ce cas, la gestion forestière n'est pas affectée par le risque et les préférences.

De manière générale, il est montré que l'aversion au risque et les préférences intertemporelles ont des effets distincts, que ce soit sur la gestion forestière, sur la prise en compte du changement climatique ou sur l'allocation des terres. Ces résultats prouvent la pertinence des préférences récursives dans l'étude de ce type de problèmes.

Contributions et limites

Par rapport à la littérature existante, la contribution majeure de cette thèse est de considérer de manière générale la gestion d'une forêt à plusieurs classes d'âge dans un contexte de risque climatique. Les résultats proposent une caractérisation claire des équilibres stationnaires en termes de gestion forestière et d'allocation des terres, en lien avec les résultats de Salo and Tahvonen (2002b) et Salo and Tahvonen (2004) obtenus en contexte déterministe. En particulier, des résultats analytiques jusqu'alors manquants sont proposés et des résultats numériques originaux viennent clarifier certains points, en particulier en ce qui concerne le rôle de certains déterminants tels que les préférences du producteur. L'usage généralisé des préférences récursives permet par ailleurs de caractériser les rôles distincts et conjoints de l'aversion au risque et des préférences intertemporelles.

Cependant, la portée de ces travaux est limitée à plusieurs égards. Tout d'abord, les résultats analytiques obtenus facilitent l'interprétation des résultats numériques mais ne définissent pas en général de formules analytiques tractables permettant de caractériser les états stationnaires. Il paraît néanmoins difficile de résoudre analytiquement le problème de la gestion forestière en contexte de risque dans le cas général considéré dans cette thèse. Toute la difficulté de cette approche réside dans la prise en compte des ajustements futurs que pourrait faire le producteur sur la base de l'observation des réalisations climatiques et ce, pour un horizon temporel infini.

Toutefois, dans un cadre plus restreint, des outils analytiques potentiellement tractables pourraient être utilisés. Par exemple, dans le cas où les préférences sont représentées par une espérance d'utilité et que l'utilité est une fonction quadratique (des variables d'état), le programme se ramène à un contrôle stochastique linéaire quadratique (*Linear Quadratic Stochastic Control*). Dans ce cas, la fonction de valeur est également une fonction quadratique des variables d'état dont les paramètres peuvent être déterminés grâce à l'équation de Bellman. Cependant, les fonctions d'utilité quadratiques sont un cas particulier et restreignent l'étude à des préférences pour lesquelles l'aversion absolue au risque est croissante (IARA) et dont l'existence réelle est discutable.

Une deuxième limite importante concerne cette fois le volet numérique. Elle est liée aux problèmes de dimensionnalité (*the curse of dimensionality*) rencontrés dans les modèles de programmation dynamique. Les temps de calcul liés à la résolution de ce genre de programme augmentent en effet exponentiellement en fonction de la dimension de l'ensemble des états possibles, qui dépend en l'occurrence du nombre de classes d'âge représentées.

L'existence de moyens informatiques de plus en plus puissants répond partiellement à ce problème et il existe des marges d'amélioration par rapport aux modèles présentés dans cette thèse. Toutefois, cette marge est à l'heure actuelle réduite et ne permet pas d'envisager des modèles numériques basés, par exemple, sur un pas de temps annuel. Ainsi, il serait opportun de développer des méthodes permettant de contourner ce problème de dimensionnalité. Par exemple, une fonction de valeur obtenue grâce à un modèle de programmation dynamique pourrait être interpolée sur une partition plus fine de l'échelle des âges.

Enfin, au delà des limites méthodologiques rencontrées, une remarque de fond peut également être formulée. Les modèles utilisés dans cette thèse présupposent en effet une gestion en futaie régulière. La gestion en futaie régulière est de fait très répandue, notamment en ce qui concerne les forêts de résineux (forêt des Landes de Gascogne, Scandinavie, etc.), ce qui justifie l'usage de tels modèles. Toutefois, les résultats discutés dans la section 2.7 suggèrent qu'en présence d'aversion au risque ou de préférences intertemporelles, les états stationnaires pourraient en réalité être des futaies irrégulières. Les travaux de thèse présentés dans ce manuscrit sont donc conditionnés à l'hypothèse que la gestion en futaie régulière est économiquement pertinente.

Perspectives

En réponse à la remarque précédente, il pourrait être intéressant d'étudier la question du risque dans un modèle de gestion en futaie irrégulière ou dans un modèle où le choix entre futaie régulière et futaie irrégulière est endogène.

Un modèle de futaie irrégulière prend en compte les effets d'interaction entre les différentes classes d'âges résultant de la compétition entre les arbres pour l'accès aux ressources (lumière, eau, nutriments). Ces effets d'interaction sont modélisés à travers des matrices de transition définissant les dynamiques de croissance des différentes classes d'âge (voir Tahvonen (2009)). Dans ce cas de figure, les états stationnaires sont des futaies irrégulières jardinées [5] comptant potentiellement plusieurs âges de récolte. Les récoltes sont réalisées à différents âges de sorte que les processus de renouvellement et de croissance de la futaie

5. Les futaies irrégulières sont des forêts dans lesquelles des arbres de différents âges se partagent un même espace (à l'échelle de la parcelle). Une futaie irrégulière est jardinée lorsqu'elle est gérée de manière à avoir une structure constante et à générer un revenu constant au cours du temps.

irrégulière soient optimisés.

Les résultats présentés dans la section 2.4.2 montrent que la présence de risque et de préférences (aversion au risque comme préférences intertemporelles) conduit également à des états stationnaires à plusieurs âges de récolte et ce, sans effets d'interaction. Il pourrait donc être intéressant dans un modèle de futaie irrégulière d'identifier les effets respectifs et combinés du risque et des effets d'interaction sur la structure forestière.

Cette thèse a porté exclusivement sur la détermination des états stationnaires. L'autre grand volet de la gestion forestière en présence d'un risque de perturbation concerne les dynamiques transitoires de la forêt quand celle-ci n'est pas à l'équilibre stationnaire. L'étude des dynamiques transitoires est d'autant plus pertinente que la probabilité de la perturbation est élevée et que son potentiel destructif est fort, car ainsi la forêt n'a que peu de chances d'atteindre un équilibre stationnaire.

En particulier, il serait intéressant d'étudier les dynamiques de reconstitution de la forêt et du revenu forestier à la suite d'une perturbation, et d'identifier dans ce cas le rôle de certains déterminants tels que l'aversion au risque ou les préférences intertemporelles. Les modèles numériques présentés dans cette thèse peuvent être utilisés pour déterminer ces dynamiques transitoires.

Une autre question importante concernant les dynamiques transitoires porte sur l'existence d'effets d'irréversibilité à la suite d'une perturbation. Des effets d'irréversibilité sont observés lorsque plusieurs états initiaux n'aboutissent pas au même état stationnaire, toutes choses égales par ailleurs. Ces effets d'irréversibilité existent dans un contexte déterministe (voir Salo and Tahvonen (2002b)) mais leur étude est d'autant plus pertinente en présence d'un risque de perturbation. L'existence de coûts fixes (sur les récoltes ou les plantations) entravant certaines transitions forestières pourraient par exemple être sources d'irréversibilité. Il est en outre très probable que les préférences jouent un rôle dans ces phénomènes.

Ensuite, tous les modèles utilisés dans cette thèse décrivent le comportement d'un unique producteur forestier (ou d'un propriétaire terrien) mais ils pourraient être utilisés dans le perfectionnement de modèles macroéconomiques, par exemple des modèles de production à l'échelle du massif forestier. Dans ce cas, l'utilisation de l'approche microéconomique décrite dans cette thèse, au sein d'un modèle macroéconomique de programmation mathématique, nécessiterait de connaître la distribution des préférences au sein de la population de producteurs considérée. Or cette distribution est généralement inconnue. Néanmoins, ces préférences pourraient en principe être révélées grâce à des modèles économétriques structurés par notre modèle microéconomique, mais cela requerrait d'importantes données sur la gestion forestière.

De ce point de vue, les perturbations en forêt, en particulier les tempêtes, sont l'occasion de recueillir une information importante sur la gestion forestière mais aussi sur les changements d'usage des sols. D'une part, ces tempêtes sont source de variabilité dans le système forestier dont les dynamiques de production sont en temps normal très lentes. D'autre part, cela pousse les organismes en charge des statistiques forestières à recueillir davantage de données qu'à la normale. Ce fut le cas, par exemple à la suite de la tempête Klaus qui a touché le massif des Landes de Gascogne en 2009.

Par ailleurs, ce type de données peut également permettre de répondre à certaines questions qui portent plus particulièrement sur les perturbations. Ainsi, la survenue d'une perturbation peut être à l'origine de certains effets qui lui sont propres, ce qui est le cas lorsque les conséquences de la perturbation sont irréversibles, ou être simplement un accélérateur révélant une tendance de fond qui se serait produite lentement en son absence. Par exemple, l'abandon de surfaces forestières au profit d'un usage alternatif peut être dû à une tempête si la destruction de la forêt est irréversible au regard des coûts réels et d'opportunité qui incomberaient à sa reconstitution. Mais elle peut aussi être progressive par la non-replantation systématique des parcelles forestières parvenues à maturité, dans ce cas, la tempête accélère simplement le processus. La réponse à ce genre de questions est importante car elle peut conditionner certaines politiques publiques, par exemple les politiques de soutien au secteur forestier.

Enfin, de manière générale, cette thèse a abordé le problème de la gestion forestière uniquement d'un point de vue positif. Pourtant, les outils numériques développés dans ce cadre pourraient aussi être utilisés dans un but normatif. Ils pourraient en effet aider certains acteurs publics (L'Etat, les collectivités locales) ou privés (acteurs de la filière bois, assureurs) à mieux comprendre les liens qui existent entre la mise en oeuvre de certains instruments réglementaires, fiscaux ou contractuels, et les décisions prises par les producteurs.